This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

NEDL TRANSFER

HN 2711 P

Purchase Henkels Sale
Nov. 27, 1908.

Joseph B Cooper Jr

Near Camden

New Jersey

12th mo 20th 1844

D KD 35266

188

THE

ARITHMETICAL EXPOSITOR;

OR,

A TREATISE

ON THE THEORY AND PRACTICE

OF

ARITHMETIC.

SUITED TO THE COMMERCE OF THE UNITED STATES.

IN TWO PARTS.

PART I.—CONTAINING ALL THE MOST USEFUL PRACTICAL RULES OF THE SCIENCE, ILLUSTRATED BY A SUFFICIENT COLLECTION OF EXAMPLES.

PART II.—CONTAINING THE LESS USEFUL PRACTICAL RULES, WITH THEIR EXEMPLIFICATIONS; A GENERAL VIEW OF THE THEORY; AND INDICATIONS OF THE MODES BY WHICH THE MORE DIFFICULT QUESTIONS IN THE WORK ARE SOLVED.

BY ENOCH LEWIS.

SECOND EDITION.

PHILADELPHIA:
KIMBER AND SHARPLESS, 8, SOUTH FOURTH ST.
ADAM WALDIE, PRINTER.
1829.

HARVARD
UNIVERSITY
LIBRARY
047172

Eastern District of Pennsylvania, to wit:

L. S. Be it remembered, That on the sixteenth day of October, in the forty-ninth year of the Independence of the United States of America, A.D. 1824, Kimber & Sharpless, of the said district, have deposited in this office the title of a book, the right whereof they claim as proprietors, in the words following, to wit:

"The Arithmetical Expositor; or, a Treatise on the Theory and Practice of Arithmetic, suited to the Commerce of the United States. In two Parts. Part I.—Containing all the most useful practical Rules of the Science, Illustrated by a sufficient Collection of Examples. Part II.—Containing the less useful practical Rules, with their exemplifications; a general view of the Theory; and Indications of the Modes by which the more difficult Questions in the work are solved. By Enoch Lewis."

In conformity to the act of the congress of the United States, intituled, "An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies, during the times therein mentioned:"—and also to the act, entitled, "An act supplementary to an act, entitled, 'An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies during the times therein mentioned,' and extending the benefits thereof to the arts of designing, engraving, and etching historical and other prints."

D. CALDWELL,
Clerk of the Eastern District of Pennsylvania.

THE
ARITHMETICAL
EXPOSITOR.
PART I.

KD35266

CONTENTS.

	Page
Introduction, - - - - -	13
Definitions, - - - - -	ib.
Notation and Numeration, - -	15
Addition of Integers, - - -	17
Subtraction of Integers, - -	21
Multiplication of Integers, -	24
Division of Integers, - - -	30
Reduction, - - - - -	38
Compound Addition, - -	50
Compound Subtraction, - -	53
Compound Multiplication, -	56
Compound Division, - -	61
The Single Rule of Three, or Simple Proportion, - - -	63
The Double Rule of Three or Compound Proportion, - -	69
Practice, - - - - - -	72
Tare and Tret, - - - -	76
Interest, - - - - - -	79
Rebate or Discount, - - -	82
Insurance, Commission and Brokage, - - - - - -	84
Equation of Payments, - -	85
The Rule of Three and Practice exemplified by Questions respecting the *Bartering* of goods and the *Loss* or *Gain* on Mercantile Transactions, - -	86
Vulgar Fractions, - - -	88
Reduction of Vulgar Fractions,	91
Addition of Vulgar Fractions -	101
Subtraction of Vulgar Fractions,	103
Multiplication of Vulgar Fractions,	104
Division of Vulgar Fractions, -	105
The Single Rule of Three in Vulgar Fractions, - - - -	105
The Double Rule of Three in Vulgar Fractions, - - - -	107
Decimal Fractions, - - - -	108
Addition of Decimals, - -	109
Subtraction of Decimals, - -	110
Multiplication of Decimals, - -	111
Division of Decimals, - -	112
Reduction of Decimals, - -	115
The Single Rule of Three in Decimals, - - - - - -	118
The Double Rule of Three in Decimals, . - - - -	120
Fellowship, - - - - -	121
Exchange, - - - - - -	126
Domestic Exchange, - -	127
Foreign Exchange, - - -	129
Arbitration of Exchange, - -	137
Involution—or the Raising of Powers, - - - - - -	141
Evolution—or the Extraction of Roots, - - - - - -	143
The Square Root, - - -	144
The Cube Root, - - - -	146
Extraction of Roots in general -	150
Duodecimals, - - - - -	154
Addition and Subtraction of Duodecimals, - - - - -	ib.
Multiplication of Duodecimals,	155
Tonnage of Ships, - - -	156
Single Position, - - -	157
Double Position, - - -	159
Promiscuous Examples, - -	162

HARVARD
UNIVERSITY
LIBRARY
047*172

PREFACE.

THE history of science, like the history of nations, is generally, in its early stages, involved in obscurity. It is the majestic river, not the streamlet that feeds it, that can be traced on the map of the world. Though we cannot now determine at what time, or by what people, the science of Arithmetic was first cultivated, it is obvious, that some knowledge of numbers must have been nearly coeval with the existence of man, and that some rude essays towards computation must have been blended with the early efforts of commercial adventure.

The foundations of science, as well as of art, were laid in our wants; and notwithstanding the advanced state in which the science of numbers is presented to us, we may still discover evident traces of its simple origin. The most natural mode of expressing small numbers, where language is wanting, is to exhibit a corresponding number of fingers; and as a number greater than *ten*, can be thus expressed, only by repeating the circuit, the number *ten* has been peculiarly marked in most languages, as well as in most systems of notation.

In most languages, numbers not exceeding ten, are designated by particular names; and the same terms, variously modified and combined, are, with a few exceptions, used to indicate all superior numbers. A process somewhat analagous, has been adopted in most systems of notation. The Hebrews, and after them the Greeks, appear to have used the first nine letters of their alphabet to denote the corresponding numbers; the second nine to indicate a number of *tens;* and to express hundreds, thousands, &c. other letters, differently marked, were used. In our present admirable notation *ten* characters are capable of denoting all possible numbers.

Strabo, who lived during the reign of Augustus, informs us that the invention of Arithmetic was then ascribed to the Phœnicians. As these people were among the first who applied themselves to navigation, and rendered it subservient to commerce, it is probable that Arithmetic was *improved* by them; but the science appears to have been known at a much earlier period to the Chaldeans and Egyptians.

According to Josephus, the knowledge of Arithmetic was communicated to the Egyptians by the patriarch Abraham, who would appear to have borrowed it from his countrymen the Chaldeans.

In Greece, the time of Thales is an era in the history of science. That eminent philosopher travelled a long time in Egypt and India, enriching his mind with such scientific knowledge as those countries afforded, and returning to his native city, he established the celebrated Ionian school, which diffused a taste for science amongst the inhabitants of Greece. During his residence in Egypt, he measured the height of the Pyramids, by means of their shadows; which may be considered as the earliest instance on record, of the application of arithmetic to geometry.

Among the disciples of Thales was the celebrated Pythagoras, who, like his master, travelled into India and Egypt, and spent two-and-twenty years of persevering industry, in collecting the knowledge and wisdom of the east. Among the objects which claimed his attention, Arithmetic was one. The multiplication table is usually ascribed to him. To him we are likewise indebted for a knowledge of the property of right angled triangles, that the square of the longest side is equal to the sum of the squares of the other two.

How far the science of Arithmetic was advanced among the ancients, cannot be clearly ascertained; but from the mutilated remains of Grecian works on this subject, which have reached us, it appears that, be-sides the fundamental rules of Arithmetic, they were acquainted with the methods of extracting the square and cube roots; and that they understood the theory of arithmetical and geometrical progressions.

Their modes of performing their calculations must, from the nature of their notation, have been exceedingly tedious and complicated: yet their knowledge of the combinations of numbers, and of the methods of reducing ratios to their simplest forms, appears to have been accurate and extensive.

The Alexandrian school was established by Lagus, one of the successors of Alexander; and it continued to flourish during upwards of ten centuries. This seminary produced a number of eminent mathematicians, one of whom was Euclid, the author of a well known treatise on geometry; the 7th, 8th and 9th books of which, form the oldest work on arithmetic extant.

About the second century of the Christian era, a new method of notation, called the sexagesimal, was introduced; of which Ptolemy the astronomer is supposed to have been the inventor. Every unit was divided into 60 parts, and each of these parts into 60 others, &c.—and the progression in whole numbers was also made sexagesimal. From one to fifty-nine, the numbers were expressed in the usual way; and sixty, called the *sexagesima prima*, was denoted thus I′—twice sixty II′, and so on to fifty-nine times sixty—when the series was resumed as before, only sixty times sixty was expressed thus I″.—When a number less than sixty was joined to a sexagesimal, it was annexed in its proper character, thus I′V denoted sixty-five.

Fractions were expressed by placing the dash at the bottom, or on the left of the numerical letter, thus, I$_{,}$ or ′I, denoted $\frac{1}{60}$. Some traces of this notation are still visible in the divisions of the circle, and of time; in the former of which, the character, as well as the scale, is retained. Though the sexagesimal notation is commonly attributed to Ptolemy, there is reason to suppose it an eastern invention. The East Indians at the present time employ the sexagesimal division of time.

Near the middle of the seventh century, the Arabs, then a fierce and uncivilized people, made themselves masters of Egypt. The famous library of Alexandria, containing nearly 700,000 volumes, the repository of the learning and genius of antiquity, was by them consigned to the flames. The books are said to have served as fuel for heating the numerous baths of that city, during six months. The loss was irretrievable, yet those people soon afterwards began to cultivate the sciences, which they had so ignorantly endeavoured to destroy. The works of the Grecian sages were translated into the Arabic; and some treatises, the originals of which are lost, have been preserved in that language. Though the scientific discoveries of the Arabs are neither numerous nor important, they have the merit of preserving many productions of more enlightened days, and transmitting them to posterity, at a time when most other nations were sunk in the most profound ignorance. The method of notation now universally adopted by the nations of Europe, was borrowed from the Arabs, who, about the beginning of the eighth century, took possession of some parts of Spain. This notation possesses the important advantages of precision, conciseness, and simplicity.

If a duodecimal, instead of a decimal scale had been used, it would have been more convienent, as it would have had a greater number of aliquot parts; but the *principle* of the notation appears incapable of improvement. It is one of those happy efforts of genius, which, arriving at its end in a manner the most simple and direct, appears, to common observation, as too obvious to require an exertion of ingenuity to effect it. But it is the peculiar privilege of genius to attain its ends by the simplest of all possible means. To the extreme simplicity of our present notation we owe, in part at least, the great advancement which the moderns have made in other sciences connected with Arithmetic.

The time which was required, by every other system of notation, to attain a knowledge of this science, or to make their calculations, can now be devoted to other pursuits.

The decimal notation, though borrowed immediately from the Arabs, appears to have originated among the natives of India. Aben-Ragel, an Arabian author of the 13th cetury, affirms that the notation in question is due to the philosophers of India. Planudes, a Greek writer, who lived near the same time, confirms this account.

At what time this notation was invented is not known. If the opinion of Aben-Ragel, above noted, is correct, we must assign it a date posterior to the time of Pythagoras; as we can hardly suppose so important an invention, if then known, likely to escape his observation; or that the sages of Greece would have adhered to their old complex system, if one so simple and complete had been offered to their acceptance.

The celebrated Gerbert, afterwards Pope Silvester II., went to Spain, then under the dominion of the Arabs, to study this science, and thence diffused it among his countrymen in France, about the year 960. It appears to have been introduced into England in the course of the succeeding century.

Since the introduction of the Indian notation, the science of Arithmetic has been improved by numerous writers: among whom Purbach, Regiomontanus, and Wallis, hold a distinguished place.* But the greatest improvement in the more abstruse arithmetical calculations, was made by Baron Napier, by his celebrated invention of logarithms. The work in which this invention was announced was published in 1614. Though logarithms are seldom noticed in treatises on Arithmetic, they are properly a part of the science, and their invention, especially as improved by Briggs, may be considered as the completion of the system. It is probable that little now remains but to render the acquisition of this branch of knowledge as easy as possible. The paths of the science may perhaps be smoothed, but it is not probable that its bounds can be further enlarged.

---

* Purbach and his pupil Regiomontanus, may be considered as having first introduced decimal fractions, in a work begun by the former, and completed by the latter about the year 1464. They were used by Ramus in his Arithmetic, about 1550, as they had previously been by the English writers, Buckley and Record. But the first who wrote expressly on the subject, was Stevinus, in "La Practique d'Arithmetique," about the year 1582. Dr. Wallis appears to have been the first, who distinctly treated of circulating decimals.

With regard to the following work, a few observations may suffice. In the business of teaching the sciences connected with this subject, the author had frequent occasion to regret the want of a work entirely adequate to the demands of his pupils. He was therefore induced to undertake this task, with a hope that his experience might qualify him to supply, at least partially, the existing deficiency.

Endeavours have been used to render the principles of this science perspicuous, and, by particular attention to accuracy in the definitions and rules, to lay a firm foundation for an acquaintance with the higher branches of knowledge.

In the arrangement of the work care has been taken to adopt an order which appeared easy and natural, and to preserve a regular dependence in the successive parts. As the money of the United States is the most convenient in practice, and the only kind used in the banks and public offices in this country, the examples relating to pecuniary transactions are mostly given in dollars and cents.

In the article on the subject of Proportion, the useless and perplexing distinction of *direct* and *inverse* proportion, has been totally discarded, and a single rule, which is simple and general, adopted.

The subject of Exchange has obtained the particular attention of the author. The intrinsic values of foreign coins, compared with those of the United States, have been stated with the utmost accuracy attainable, from the authorities within the knowledge of the writer.

The work was divided into two parts, with a view of meeting the wants of different classes of learners. To those who have not leisure or inclination to acquire a general acquaintance with the subject, part first may be thought sufficient.

Part second was designed to complete what was wanting in the first; and, in order to render the principles on which the different operations are founded as obvious as possible, a general view of the theory has been subjoined.

The indications of the modes by which the more difficult questions contained in the work may be solved, are intended to assist such as may study the science without the aid of a teacher; and thereby to obviate the necessity of a key.

PREFACE TO THE SECOND EDITION.

This second edition has undergone an attentive revision; and the errors which had found their way into the first have been carefully corrected. In the definitions and rules no important change has been made. The number of examples, to exercise the student, has been considerably increased, and in these, the original plan of interweaving other instruction with arithmetical exercises, has been kept in view. The former arrangement is still preserved, as it appears questionable whether any change in this respect could be advantageously made. Perhaps the subject of vulgar and decimal fractions might, with propriety, be introduced in an earlier part of the work. If the attention of the pupil was directed to these parts of the science previously to the rule of proportion, the examples integral and fractional, in the single and double rule of three, might be promiscuously solved.

If any teachers should prefer this course, they will experience no difficulty in its adoption, as the rules will be found very nearly as well adapted to that arrangement, as to the one which has been preferred.

It is certainly desirable to avoid all needless distinctions. And it must be admitted that the division of proportion into separate heads is not strictly scientific; the division, however, could not be avoided, without subjecting the student to the probability of encountering greater difficulties in the early stages of his progress than by the usual arrangement.

THE
ARITHMETICAL EXPOSITOR.

INTRODUCTION.

ARITHMETIC is the science of computation. It contains the doctrine of the powers and properties of numbers; and is divided into *specious* and *numeral.*

Specious Arithmetic, commonly called Algrebra, is the art of computing by symbols. It is principally used in the investigation of general rules, or the solution of such questions as are too complex to be conveniently managed by simple numbers. Many of the rules used in Numeral Arithmetic are founded on it.

Numeral Arithmetic is that division of the general science which is confined to operations performed with the common numerical characters or digits.

DEFINITIONS.

1. A Unit or Integer is an arbitrary quantity or individual thing, either real or imaginary, considered in an undivided capacity; and is the measure to which all other quantities of the same kind are referred.

2. An Applicate Unit is that which relates, or is applied to, some particular thing or species of things; as *one* dollar, *one* gallon, *one* man, &c.

3. An Abstract Unit is that which refers to no particular object, but is taken in a general sense.

4. Number or Multitude is an assemblage of units; and is either *abstract* or *applicate*, according to the quality of its component units.

5. An Integral or Whole Number is composed of units considered without subdivision.

6. Compound Numbers are those which result from the division of applicate units, for practical purposes, into other inferior denominations.

7. When one number is contained in another a certain number of times exactly, the number *contained* is said *to measure* the containing number. The measuring number

is called a *part* or *aliquot part* of the number measured. The measured number is called a *composite number*, as being composed of the numbers which measure it. It is also termed a *multiple* of its aliquot parts.

8. When a less number is not contained any number of times exactly in a greater, the former is termed an *aliquant part* of the latter.

9. A number, which has no measure or aliquot part except the unit, is termed a *prime number*.

10. A number, which measures two or more other numbers, is called their *common measure:* and the measured numbers are said to be *composite* to each other.

11. Two numbers which have no common measure, are said to be *prime to each other*.

12. If one number is measured by two or more other numbers, it is called their *common multiple*.

13. An *even number* is a multiple of the number two.

14. An *odd number* is that which is not measured by the number two.

15. A *perfect number* is equal to the sum of all its aliquot parts.

The ten numerical characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are called *digits*.* The last nine are termed *significant figures*, because they have certain values assigned them, though standing alone, The first (0), is called a *cipher*† or *insignificant figure*, its value when alone being nothing.

Note.—The following characters, though properly a part of the language of Algebra, are sometimes introduced with advantage into common arithmetic.

SIGNS.	SIGNIFICATIONS.
=	equal, as 100 *cents*=1 *dollar*.
+	more, or increased by, as $4+3=7$.
—	less, or diminished by, as $5-2=3$.
×	into, or multiplied by, as $7\times6=42$.
÷	by, or divided by, as $15\div5=3$.
: :: :	as, is to, so is, to; as 2: 5 : : 6 : 15, that is, as 2 is to 5 so is 6 to 15.

---

* From *digitus*, a finger, their number being equal to that of the fingers on both our hands.

† From *tsapher*, (Arabic,) a void or empty space. Hence also our English zero.

Arithmetic consists of the following fundamental divisions: viz. Notation, Numeration, Addition, Subtraction, Multiplication, and Division.

NOTATION AND NUMERATION.

These terms, though frequently used without distinction, are not synonymous.

Notation teaches to express, by combinations of the numerical characters, any number proposed.

Numeration teaches to read in words any given combination of figures.

In order to adapt the numerical characters to the expression of numbers universally, a general and a local value have been assigned to each figure. The former is that which it possesses when standing alone. The latter depends upon its combination with others. Thus in the numeration of integers, beginning at the right, the first figure is called *units*, the second *tens*, the third *hundreds*, and so on, increasing their local values in a tenfold proportion; as exemplified in the following

TABLE.

Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units	
								9	nine.
							8	9	eighty-nine.
						7	8	9	seven hundred and eighty-nine.
					6	7	8	9	6 thousand 789.
				5	6	7	8	9	56 thousand 789.
			4	5	6	7	8	9	456 thousand 789.
		3	4	5	6	7	8	9	3 million 456 thousand 789.
	2	3	4	5	6	7	8	9	23 million 456 thousand 789.
1	2	3	4	5	6	7	8	9	123 million 456 thousand 789.

Units are sometimes said to be of different orders according to their situation; thus beginning at the right, the first figure denotes units of the first order; the second, or the tens, units of the second order; the third, or the hundreds, units of the third order, &c.—Three orders constitute a period. This is rendered very simple by the following table.

PERIODS.

IX.	VIII.	VII.	VI.	V.	IV.	III.	II.	I.
Hundreds, Tens, Units,	Hundreds, Tens, Units,	Hundreds, Tens, Units,	Hundreds, Tens, Units,	Hundreds, Tens, Units,	Hundreds, Tens, Units,	Hundreds, Tens, Units,	Hundreds, Tens, Units,	Hundreds, Tens, Units,
Septillions, &c.	Sextillions.	Quintillions.	Quadrillions	Trillions.	Billions.	Millions.	Thousands.	Units.

This is the French notation, which is here introduced on account of its simplicity. The English, after the first nine figures, is somewhat different; Millions, Billions, &c. including six figures each. Thus,

Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.
368749,	568356,	863257,	986241,	567379,	324654,

Trillions.	Billions.	Millions.	Thousands.	Units.
832569,	876371,	428763,	321,	573.

EXAMPLES.

In figures express the following numbers:

1. Two hundred and seventy-five. *Facit* 275.
2. Three hundred and twenty. *Facit* 320.
3. Four thousand six hundred and twenty-seven.
4. Five thousand four hundred and fifty-two.
5. Twenty-five thousand three hundred.
6. Eighty-seven thousand and forty-six.
7. Two hundred and eighty-five thousand seven hundred and nineteen.
8. Seven hundred and thirty-six thousand one hundred and fifty-six.

9. Seven million one hundred and sixty-one thousand nine hundred and six.

10. Three million seven hundred thousand six hundred and seventy-four.

11. Twenty-seven million fifty four thousand three hundred and ninety-six.

12. One hundred and eighty-two million three hundred and seventy-five thousand nine hundred and nine.

Numbers may also be expressed by letters, called *Numerical Letters* or *Roman Numbers;* which are, with the numbers they represent, as follow :

1	5	10	50	100	500	1000
I,	V,	X,	L,	C,	D,	M.

A letter of less value standing before a greater diminishes, but when placed after increases the value of the greater. By thus combining the numerical letters any number may be expressed.

1824
MDCCCXXIV.

ADDITION OF INTEGERS.

The business of addition is to collect, into one sum, several given quantities; which, in applicate numbers, must always be of the same kind, or measure quantities of like species.

RULE.

Set the given numbers under each other, with units under units, tens under tens, &c. then beginning at the right hand, add the several numbers in the first column, and reserving the tens, set down the remaining units directly under said column ; add the tens thus reserved, reckoning them as units, with the numbers in the next column ; set down and carry as before, and so proceed to the last or left hand column, under which write the whole amount.

PROOF.

1. Perform the operation a second time agreeably to the

rule, but in one case begin at the bottom, in the other at the top. Or,

2. Reserve one or more of the given numbers, find the sum of the rest, and thereto add the number or numbers reserved, the result will be equal to the sum found by the rule. Or,

3. Add all the digits in the several given numbers together, either sidewise or vertically, rejecting nine as often as it occurs, the result, if the operations are rightly performed, will correspond to that obtained, by a like process, from the whole sum.

N. B. These results will sometimes correspond when the operations have not been correctly performed.

EXAMPLES.

4732	56425	236487
2543	15972	684365
6728	87236	142673
5914	69184	803527
19917		

542394	3765729	859347
186726	842567	167285
625632	34634	762942
916269	526906	865729
84357	7465723	432736
5624	873946	743653

523489	2763542	7253867
347237	8254728	8347952
524672	1475369	1736415
832754	7527457	2543625
476511	7236458	3857244
652763	2745242	2763158
475328	7524631	9374696

5234572	9465342	53484326
6345675	1754657	47262936
4567467	8324758	72846427
8645736	7536462	54734984
6765427	1645768	46515674
3654324	7134231	52737063
2432535	2786352	16042463

APPLICATION.

1. Collect into one sum, three thousand four hundred and twenty-seven, six hundred and fifty-two, seven thousand nine hundred and forty-six, and thirteen thousand nine hundred and forty-six. *Facit* 25,929.

2. If from Adam to Seth was 130 years; from Seth to Enos 105; from Enos to Cainan 90; from Cainan to Mahalaleel 70; from Mahalaleel to Jared 65; from Jared to Enoch 162; from Enoch to Methuselah 65; what was the age of Adam at the birth of Methuselah? *Ans.* 687.

3. The distance from New-York to Philadelphia is 95 miles, thence to Baltimore 102, and from the latter to Richmond 176; how far is it from New York to Richmond? *Ans.* 373 *miles.*

4. The number of inhabitants in the eastern parts of the United States in the year 1800 was as follows: New Hampshire 183858; Massachusetts 573565; Connecticut 251002; Rhode Island 69122; Vermont 154465. Quere their amount. *Ans.* 1232012.

5. From the Christian era to the council of Nice was 325 years, from that time to the birth of Mahomet 244, thence to the first crusade 520, from that to the discovery of America 403, thence to the declaration of American Independence 284; and from that time to the Abolition of the African slave trade 31. In what year of the Christian era did this last event take place? *Ans.* 1807.

6. According to a census taken in the year 1800, there

were in Pennsylvania and the New England states 3045 slaves; in New York and New Jersey 33035; in Delaware and Maryland 113860; in Virginia 346968; in Kentucky and Tennessee 53927; in North Carolina 133196; in South Carolina and Georgia 205850. How many were there in all? *Ans.* 889881.

7. If from the birth of Shem to that of Arphaxad was 101 years, thence to that of Salah 36, thence to the birth of Eber 31; from that time to the birth of Peleg 35, thence to that of Reu 31, thence to Serug 33, thence to Nahor 31, thence to Terah 30, thence to the birth of Abram 130; how old was Shem at the birth of Abram?

Ans. 458 *years.*

8. The following is a statement of the quantity of strong beer brewed by the twelve principal houses in London in the course of a year. Barclay 264105 *barrels*, Meux 220094, Hanbury 142179, Whitbread 122316, Calvert 105887, H. Meux 103152; the remaining six 380445. Quere the whole amount.

Ans. 1338178.

9. Suppose Europe contains 4456065 square miles, Asia 10768823, Africa 9654807, America 14110874, seas and unknown parts 160522026. How many are contained in the whole world?

Ans. 199512595 *square miles.*

10. The census for the year 1820 gives the inhabitants of the United States as follows: Maine 298335, New Hampshire 244161, Massachusetts 523287, Rhode Island 83059, Connecticut 275248, Vermont 235764, New York 1372812, New Jersey 277575, Pennsylvania 1049398, Delaware 72749, Maryland 407350, Virginia 1065366, North Carolina, 638829, South Carolina 490309, Georgia 340989, Alabama 127901, Mississippi 75448, Louisiana 153407, Tennessee 422813, Kentucky 564317, Ohio 581434, Indiana 147178, Illinois 55211, Missouri 66586, Territory of Michigan 8896, Arkansas 14273 ,District of Columbia 33039. Required the number in all.

Ans. 9625734.

11. According to Seyberts, the contents of the following states in square miles are—New Hampshire 9491 ; Vermont 10237 ; Massachusetts and Maine 46250 ; Rhode

Island 1580; Connecticet 4674; New York 45000; New Jersey 8320; Pennsylvania 46800; Delaware 2120; Maryland 14000; Virginia 70000; North Carolina 48000; South Carolina 24080; Georgia 62000; Kentucky 50000; Tennessee 40000; Ohio 39128. What is the whole amount? *Answer* 521680.

12. In 1817, the number of emigrants who arrived in ten of our ports was as follows: Boston 2200; New York 7634; Perth Amboy 637; Philadelphia 7085; Wilmington 558; Baltimore 1817; Norfolk 520; Charleston 747; Savannah 163; New Orleans 879. How many were there in all? *Answer* 22240.

13. It appears by the census of 1820, that Philadelphia (between Vine and Cedar streets) contained 63802 inhabitants; the Northern Liberties 19378; Kensington 7118; Spring Garden 3498; Penn Township 3105; Southwark 14713; Moyamensing 3963. What was the whole number? *Answer* 115577.

14. The inhabitants of the British Islands in 1821 were stated thus: England 11260555; Wales 717103; Scotland 2092014; Ireland 6846949; army and navy 310000; Isle of Man 40081; Guernsey and its dependent islands 20827; Jersey 28600; Scilly Isles 2614. What was the whole number? *Answer* 21318743.

15. The British dominions are estimated in square miles as follows: British Islands 125240; Hanover 14600; British India 553000; Cape of Good Hope 120000; inhabited dominions in North America 240000; British West Indies 9000; Guiana 10000. Quere the whole amount.

Answer 10718401.

SUBTRACTION OF INTEGERS.

Subtraction is the rule which teaches to take a less number from a greater. In every example belonging to this rule, there are three parts or terms.

1. The minuend or number to be diminished.
2. The subtrahend or number to be subtracted.
3. The remainder or difference.

These terms must be all abstract numbers; or applicate numbers of the same species.

RULE.

Set the subtrahend under the minuend, with units under units, tens under tens, &c. Then beginning at the right hand, take, if possible, the several numbers in the subtrahend from those directly above, and set the remainders below them: but when the figure to be subtracted has a greater value than that above it, add ten to the latter, subtract the former from the sum, add one to the next subtrahend figure, and so proceed.

PROOF.

1. Add the remainder and subtrahend together, the sum will be equal to the minuend. Or,

2. Subtract the remainder from the minuend, the result will be equal to the given subtrahend. Or,

3. Collect the digits in the several terms rejecting the nines, the result of the subtrahend subtracted from that of the minuend, borrowing nine if necessary, will leave a result equal to that of the remainder.

EXAMPLES.

From	423652	187652	543214523	2
Take	132941	92461	135724635	0
Rem.	290711	95191	407489888	2
Proof.	423652	92461		

From	3621531	17362513	84256723
Take	1847267	9834674	52873964

APPLICATION.

1. From forty-five thousand three hundred and twenty-two, take twenty-four thousand five hundred and seventeen.
Facit 20805.

2. Borrowed three thousand two hundred fifty-six dol-

lars, and paid in part one thousand six hundred thirty-four, how much remains unpaid? *Ans.* 1622.

3. It is supposed that Solomon finished the temple in the year of the world 2999, and that it was destroyed by the Chaldeans in the year 3416, how long then did it stand? *Ans.* 417 *years.*

4. Near the close of the eighteenth century the population of the British West Indies was estimated at 455684 blacks, and 65305 whites. Quere their difference. *Ans.* 390379.

5. William Penn, founder of Pennsylvania, was born in the year 1644 and died in 1718, what was his age? *Ans.* 74 *years.*

6. By the census taken in 1800 the inhabitants of the United States were 5305666, and in 1810 they amounted to 7239903: required their increase during the ten intervening years. *Ans.* 1934237.

7. The mariner's compass was invented about the year 1302; and America was discovered in 1492: how long did the former precede the latter? *Ans.* 190 *years.*

8. The art of printing is said to have been invented in the year 1440, and the reformation was commenced by Martin Luther about the year 1517: how many years intervened? *Ans.* 77.

9. The Arabian or Indian method of notation is supposed to have been brought into France about the year 960, and the science of fluxions was invented by Newton in 1665, what was the interval? *Ans.* 705 *years.*

10. The inhabitants of the United States in 1810, were 7239903, and in 1820, their number was 9625734: required the increase during these ten years. *Ans.* 2385831.

11. The diameter of the sun is computed to be 883246 miles; and the diameters of the primary planets as follow: Mercury 3224, Venus 7687, the earth 7958, Mars 4189, Jupiter 89170, Saturn 79042, Herschel 35112, Ceres 1624, Pallas 2099, Juno 1425, Vesta 237 miles: how much does the diameter of the sun exceed the sum of the diameters of the planets? *Ans.* 651479 *miles.*

12. In the year 1790, there were in the United States 697697 slaves, and in 1820, they amounted to 1531436, required their increase during these thirty years. *Ans.* 833739.

13. The use of spectacles is supposed to have been discovered by Roger Bacon, about the year 1274; and Galileo formed his telescope in 1609: required the interval.
Ans. 335.

14. In 1821, the taxable inhabitants of Pennsylvania amounted to 208439; and in 1828, their number was 243,874: what was the increase? *Ans*. 35435.

15. In the year 1820, the white males in the U. States amounted to 3995073, and the white females to 3866657: quere the difference? *Ans*. 128416.

16. The whole population in the state of New York, in 1810, was 959049; and in 1820, they amounted to 1372812: required the increase. *Ans*. 423763.

17. The inhabitants of New Jersey, in 1790, were 184139; and in 1820, their number was 277575: what was the interval in time and the increase of numbers?
Ans. 30 years and 93436.

18. The annual value of goods imported into the United States, about the beginning of the present century, was 75316000 dollars, and the value exported 68461000: what was the difference? *Ans*. 6855000.

MULTIPLICATION OF INTEGERS.

Multiplication is a compendious method of adding together a given number of equal numbers or quantities, or of finding a number which shall contain a given number as often as another given number contains a unit.

Multiplication consists of three parts or terms: viz. multiplicand, multiplier, and product.

The multiplicand is the number to be multiplied, which may be either an abstract or an applicate number.

The multiplier is the number denoting how often the multiplicand is to be repeated. It must always be an abstract number.

The product is the number sought, containing the multiplicand as often as the multiplier contains a unit. It is of the same species as the multiplicand.*

---

* When a product is multiplied by a second multiplier, and the

The multiplicand and multiplier are frequently termed factors.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

This table is evidently formed by addition. The several numbers in the first line added to themselves, form those in the second: and in general, the numbers in any line added to those directly over them in the first compose the correspondent numbers in the next following line.

CASE 1.

When the multiplier is a factor in the table.

---

latter product by a third, &c. the result is called the continued product of the several factors; and the original multiplicand is said to be *multiplied continually* by the multipliers which are used

RULE.

Set the multiplier under the multiplicand with units under units, &c. as in addition: then beginning at the right, multiply each figure of the multiplicand by the multiplier, set down the units, and add the tens to the next left hand product; and so proceed to the last, and there set down the whole product.

EXAMPLES.

$$\begin{array}{r} 4372543 \\ 2 \\ \hline 8745086 \end{array} \qquad \begin{array}{r} 427365 \\ 2 \\ \hline \\ \end{array} \qquad \begin{array}{r} 73194265 \\ 3 \\ \hline \\ \end{array} \qquad \begin{array}{r} 5729385 \\ 4 \\ \hline \\ \end{array}$$

$$\begin{array}{r} 8364279 \\ 5 \\ \hline \end{array} \qquad \begin{array}{r} 8634279 \\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 836573257 \\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 367954262 \\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 724357294 \\ 9 \\ \hline \end{array} \qquad \begin{array}{r} 581479354 \\ 10 \\ \hline \end{array} \qquad \begin{array}{r} 3173487567 \\ 11 \\ \hline \end{array} \qquad \begin{array}{r} 487361246 \\ 12 \\ \hline \end{array}$$

CASE 2.

When the multiplier consists of several figures.

RULE.

Set down the multiplier as in Case 1, and find the product of the whole multiplicand by each multiplying figure; place the right hand digit of each product directly under the multiplier by which it is produced: the sum of these (adding vertically) will be the product required.

PROOF.

1. Repeat the operation with the factors changed. Or,
2. Double one factor and multiply by the other, this product will be the double of the product first obtained. Or,

3. Add the digits in each factor sidewise; reject the nines; multiply the remainders and reject the nines as before; the result will be the same as that arising from the addition of the several figures in the product, the nines being in like manner rejected.

EXAMPLES.

Multiply	4267234	1		4267234
By	35	8		2
	21336170	8		8534468
	12801702	⋮	*Proof*	35
Product	149353190........	8		42672340
	2			25603404
	298706380		*Proof*	298706380

2.	Multiply	3725436 by	43	*Product* 160193748
3.	——	12765235 by	275	—— 3510439625
4.	——	537467 by	367	—— 197250389
5.	——	673526 by	2674	—— 1801008524
6.	——	852362 by	1567	—— 1335651254
7.	——	346726 by	3426	—— 1187883276
8.	——	876428 by	729216	*Prod.* 638934320448
9.	——	342754 by	84217	*Prod.* 28865713618
10.	——	94276542 by	315945	*Prod.* 29786202062190
11.	——	5637459 by	7515318	*Prod.* 42367297096962
12.	——	9647657 by	872792	*Prod.* 842039784834
13.	——	87469865 by	567763	*Prod.* 49662152961995

CONTRACTIONS APPLICABLE TO PARTICULAR CASES.

1. When there are ciphers on the right of one or both factors, omit them in the operation, and annex as many to the product.

2. When the multiplier is the exact product of two factors in the table, multiply by one of those factors and that product by the other.

3. When the multiplier exceeds 12, but is less than 20, multiply by the units, and to the several products add the next right hand figure of the multiplicand.

4. When the multiplier is 21, 31, &c. set down the units of the multiplicand; multiply by the tens of the multiplier, adding the next left hand figure of the multiplicand.

5. When two or more adjacent figures of the multiplier form a multiple of some other one; multiply the product of the latter by the number of times it is contained in the former.

6. When the multiplier consists entirely of nines; annex as many ciphers to the given multiplicand, and subtract the same multiplicand from the result.

EXAMPLES.

Multiply 476895 by 27
Contr 2. 3
1430685
9
12876165

Multiply 375289 by 17
17 *Contr.* 3.
6379913

Multiply 53674268
by 392749 *Contr.* 5.
375719876 ×7; 7×7=49
2630039132×8; 49×8=392.
21040313056
21080515082732

Multiply 4678946 by 9999
46789460000 *Contr.* 6.
4678946
46784781054

Multiply 4678947 by 51
51 *Contr.*4.
238626297

2.	Multiply	6747040	by	34500	*Facit*	232772880000
3.	——	2573624	by	15	—	38604360
4.	——	179832	by	71	—	12768072
5.	——	8423675	by	64	—	539115200
6.	——	5246789	by	81	—	424989909
7.	——	7365426	by	99	—	729177174
8.	——	84620954	by	546	—	46203040884
9.	——	47365428	by	288729	—	13675772661012
10.	——	38475694	by	999	—	38437218306
11.	——	573698	by	6993	—	4011870
12.	——	3485723	by	448756	—	1564239110588
13.	——	8965472	by	378954	—	3397501476288
14.	——	27697584	by	848432	—	23499516588288

APPLICATION.

1. In the year 1772, one hundred vessels sailed from Liverpool for the coast of Africa; now suppose each of these carried 575 slaves from that continent to the West Indies; of how many inhabitants was Africa thus despoiled by this iniquitous traffic? *Ans.* 57500.

2. Admit an orchard to consist of 126 rows, 109 trees in a row; and to have 10007 apples on each tree; how many trees and apples does the orchard contain?
Ans. 13734 *trees*, 13830138 *apples.*

3. A late Persian ambassador at the British court was said to have a family of 63 children; and the city of Ispahan, the capital of Persia, has been computed to contain an hundred thousand inhabitants: now supposing 15674 of these to be fathers of such families, required the number of their children. *Ans.* 987462.

4. If the two houses of congress contain 261 members, what is the amount of their wages per day at 8 dollars each? *Ans.* $ 2088.

5. If the wages of the members composing the two houses of congress, for one day, be 2088 dollars, what is the amount in 4 months, supposing each month to contain 30 days? *Ans.* 250560.

Note. The operation may frequently be abridged on the principle of the 3d and 4th methods, when a cipher or ciphers intervene; *e. g.* when the multiplier is 103, 401, &c. and the same method may be readily applied to other more complex cases.

6. The army of Xerxes may be supposed to have consumed 6254 barrels of flour at a meal; how many barrels would serve them a year, allowing three meals a day, and 365 days in the year? *Ans.* 6848130.

7. The earth is computed to move at the rate of 58000 miles per hour, how far does it move in a year consisting of 8766 hours? *Ans.* 508428000 *miles.*

8. There are four double forcing pumps at the Fair Mount Works on the river Schuylkill, each of which is computed to throw into the reservoir 1440000 gallons of water in 24 hours, for the use of Philadelphia—How many gallons may thus be furnished in a month of 31 days, supposing all the pumps to be in constant operation?

Ans. 178560000 *gallons.*

9. The subjects of the Chinese empire have been supposed to amount to 333000000; how many pounds of rice will they consume in a year, allowing 5 pounds a week for each person, and 52 weeks to compose a year?

Ans. 86580000000.

DIVISION OF INTEGERS.

Division is the converse of multiplication; the product and one factor being given to find the other; or it is a method of finding a number which contains a unit as many times as one given number contains another. It consists of three, or sometimes of four parts: viz. The dividend, the divisor, the quotient, and the remainder.

The *dividend* is the number to be divided, or that which *contains* the other.

The *divisor* is the number contained.

The *quotient* is the number expressing how often the divisor is contained in the dividend, or the number which contains a unit as often as the dividend contains the divisor.

The *remainder*, when there is one, is the excess of the dividend above the greatest multiple of the divisor, which it contains.

Note.—That portion of the dividend which is used for determining a single figure of the quotient, or which contains the divisor less than ten times, is termed a dividual.

Division is commonly distinguished into long and short. Long division may be used whether the divisor consists of one figure or of many.

RULE FOR LONG DIVISION.

Take for the first dividual as many only of the left hand figures of the dividend as will contain the divisor; try how often the divisor is contained in this dividual, and set down the result for the first quotient figure; multiply the divisor by this quotient; place the product under the dividual and subtract it therefrom; to the remainder annex the succeeding figure of the dividend for a new dividual, with which proceed as before.

PROOF.*

1. Multiply the quotient by the divisor, and to the product add the remainder, if any, the sum will be equal to the dividend. Or,

2. Subtract the remainder from the dividend and divide by the quotient; the result will be equal to the given divisor. Or,

3. Any multiple of the difference between the dividend and remainder, divided by a like multiple of the divisor, will produce the same quotient.

EXAMPLES.

```
3)47526(15842 quotient.
  3          3
  —      ——————
  17     47526 proof.
  15     ——————
  ——
   25
   24
   ——
    12
    12
    ——
     6
     6
```

---

* Some writers direct, to add the final remainder and the several

2.	Divide	456789	by	2	*Facit*	228298	*Rem.*	1
3.	—	369724	by	4	—	92431	—	
4.	—	1769872	by	5	—	353974	—	2
5.	—	3674573	by	7	—	524939	—	
6.	—	4356789	by	8	—	544598	—	5
7.	—	3567894	by	9	—	396432	—	6
8.	—	4256798	by	11	—	386981	—	7
9.	—	90465767	by	12	—	7538813	—	11
10.	—	35743786	by	12	—	2978648	—	10
11.	—	47635679	by	13	—	3664283	—	
12.	—	36242798	by	15	—	2416186	—	8
13.	—	547039874	by	17	—	32178816	—	2
14.	—	465672895	by	19	—	24509099	—	14

Short division is a contracted method of performing the operations, to be used when the divisor does not exceed 12, the greatest factor in the multiplication table.

RULE FOR SHORT DIVISION.

Take a first dividual as in the former rule, try how often the divisor is contained therein; set down the resulting figure, and subtract mentally the product of that figure and the divisor from the dividual, the remainder, (considered as *tens*,) with the next figure of the dividend annexed, is the succeeding dividual.

EXAMPLES.*

4)3695672   3)572596   5)3742785

quot. 923918
4

Proof 3695672

---

dividuals, paying proper regard to the order of the figures, the sum will be equal to the dividend. But this method evidently verifies the subtractions only.

* It may perhaps be found eligible, on pupils being introduced into this rule, to direct them, first to perform the operation by the former method, and then to go over it by this. The whole process being thus exhibited, the transition to the contracted mode will be rendered more easy; and the fact, that the two methods are essentially the same, familiarized to their minds.

6)3478952

8)5794678

11)5678346

12)4362573

7)84662359

9)4567898

11)6942364

9)6754286

4)9873268

10)87694230

12)67523648

8)67236598

EXAMPLES TO EXERCISE THE TWO FOREGOING RULES.

	Divide	by	Facit.	Rem.
1.	457346	2	228673	
2.	846208	3	282069	1
3.	620482	5	124096	2
4.	131519	6	21919	5
5.	891057	7	127293	6
6.	9218706	8	1152338	2
7.	1407025	9	156336	1
8.	1146376	11	104216	
9.	6374037	12	531169	9
10.	9420674	13	724667	3
11.	3271916	17	192465	11
12.	9643007	23	419261	4
13.	76919603	57	1349466	41
14.	12716479	119	106861	20
15.	27639007	157	176044	99
16.	67093794	286	234593	196
17.	34230635	3754	9118	1663
18.	91427926	4567	20019	1153
19.	142769437	73695	1937	22222
20.	9684976376	325967	29711	170839
21.	7061327109	469706	15033	236811

Note.—In the examples above wrought, the divisor is placed on the left hand of the dividend; its place however is a matter of indifference. In French books on arithmetic it is placed on the right hand with the quotient immediately below it. This method has the advantage of placing the divisor near the figures by which it is multiplied. It also occupies less room.

CONTRACTIONS APPLICABLE TO PARTICULAR CASES.

1. When there are ciphers on the right of the divisor omit them in the operation, separating as many right hand figures of the dividend, and to the remainder annex the figures so separated.

2. When the divisor is the exact product of two or more factors in the multiplication table, divide successively by those factors; if remainders occur, multiply the last remainder by the last divisor but one, and add the remainder belonging to said divisor; multiply the result by the preceding divisor, adding the correspondent remainder as before; in this manner proceed through all the preceding divisors, the last result will be the true remainder.

3. When the divisor consists wholly of nines; beginning at the right hand, distinguish the dividend into periods, of as many figures each as the divisor contains; then under each of these set down the adjacent left hand period, and repeat the process until the dividend is exhausted; add the numbers vertically, increasing the units of the first period by the number carried to the second; from the sum thus obtained separate a right hand period as before for the remainder, and the rest will be the quotient required.

N. B. If this remainder should be equal to the divisor, the quotient must evidently be increased by a unit, and the true remainder is 0.

EXAMPLES.

4674|00)6895674|36(1475 *quot. Contr.* 1.
4674

22216
18696

35207
32718

24894
23370

152436 *rem.*

Divide 41457254 by 168
4)41457254 *Contr.* 2.
6)10364313 2
7)1727385 3
246769 2
6
15
4
62 *rem.*

Divide 47|6856|4834 by 9999 *Contr.* 3.
47|6856
47
quot. 476904|1737
1
1738 *rem.*

Proof by contraction 6. Multiply 4769041738
476904
4768564834

Divide	by	Quot.	Rem.
27648748	18	1536041	10
39428697	24	1642862	9
8467965248	3400	2490578	48
972365248	243	4001503	19
398764856	999	399164	20
542896754	3600	150804	2354
728946758	2160	337475	758
643752698	9999	64381	7079
863769304	1512	571275	1504

Note.—The operations, in some instances, may be abridged, by substituting multiplication for division; and reciprocally.

Thus, to multiply by 5, annex a cipher, and divide by 2.

To multiply by 25, annex two ciphers, and divide by 4.

To multiply by 125, annex three ciphers, and divide by 8.

To multiply by 75, annex two ciphers, and subtract one fourth of the result.

To divide by 5, cut off the right hand figure for a remainder, and multiply the rest by 2 for a quotient.

To divide by 25, cut off the two right figures for a remainder, and multiply the rest by 4.

To divide by 125, cut off three figures on the right hand for a remainder, and multiply the rest by 8 for a quotient.

If in any of these cases the remainder should exceed the divisor, the quotient must be increased by the number indicating how often the divisor is contained in this remainder, and the latter diminished by the greatest multiple of the divisor which it contains.

EXAMPLES.

Multiply 439254 by 25

4)43925400

prod. 10981350

Multiply 3635916 by 76

4)362591600

90647900

prod. 271943700

Divide 865736679 by 125

865736 679

8 625=125×5

6925886 54 *rem.*

5

6925893 *quotient.*

Divide 3698342 by 25

36983 42

4 25

147932 17*rem.*

1

147933 *quotient.*

APPLICATION.

1. The number of letters in a volume being 2344125, of which 4465 were contained in a page; required the number of pages. *Ans.* 525.

2. If a cistern containing 15168 gallons be emptied in 4 hours, by 48 equal vents, what quantity is discharged by each adjutage in that time? also how much per hour, admitting the velocity of the fluid to be uniform?

Ans. 316 *galls.* and 79 *gall. per hour.*

3. The inhabitants of the United States were stated (in 1810,) at 7239903, and their territories have been estimated at 2560000000 acres of land: how many acres does this furnish for each inhabitant? *Ans.* 353+.

4. The Israelites are computed to have taken possession of Canaan about 1448 years before the Christian era, and the Jewish state was overturned by the Romans, in the year 70. How many generations passed during the interval, allowing 33 years to a generation? *Ans.* 46.

5. The population of Paris is stated at 713966 persons, of whom the clergy compose one in 612; how many of the latter description does that city contain?

Ans. 1166+.

6. The income of the Lord Chancellor of England for the year 1811, was 99280 dollars; how much was that for each day? *Ans.* 272 *dollars.*

7. If the French army which invaded Russia in 1812, consumed 17534 barrels of flour and 11698 barrels of meat in a week; what number of wagons, each carrying 16 barrels, would be required to carry the provisions consumed in a day? *Ans.* 261.

EXAMPLES OF MULTIPLICATION AND DIVISION.

1. If 279936 be multiplied by 16807, and the product be divided by 11907, what will the quotient be?

Ans. 2765952.

2. Divide the continued product of 117649, 15625 and 2187 by the product of 2401 and 729.

Result 2296875.

3. In the latter part of the year 1740 the snow is said to have fallen in the eastern parts of Pennsylvania to the regular depth of a yard; how many hogsheads of water were thus deposited on an acre of land, the acre containing 4840 yards, the solid yard of snow weighing 774 ounces, and the hogshead of water 8422? *Ans.* 444+.

4. The great pyramid of Egypt is computed to contain 1539798400 cubic yards of stone; now each cubic yard

being supposed to weigh 1406 pounds, how many ships could be loaded with the stone of that immense fabric, each ship being allowed to carry 496525 pounds?

Ans. 4360216+.

5. A square mile contains 640 acres of land; and a bushel of wheat is supposed to contain 491520 grains; how many grains of wheat would a square mile produce, each acre yielding 25 bushels? Also how many loaves of bread would this wheat make, allowing 30720 grains to a loaf?

Ans. 7864720000 *grains; and* 256000 *loaves.*

◆

REDUCTION.

Reduction is the conversion of applicate numbers of one denomination, to those of another, retaining the original value.—Or the changing of given numbers to others expressive of the same value.

RULE.

When the given numbers are to be reduced to a *lower* denomination; multiply by the number indicating how many units of the lower denomination make one of the higher.

When the given numbers are to be reduced to an *higher* denomination: *divide* by the number indicating how many units of the lower denomination make one of the higher.

When there are denominations between that in which the numbers are given, and the one to which they are required to be reduced, it is most convenient in practice to reduce them to the intermediate denominations.

When the given numbers are of divers denominations, in reducing to lower names, the inferior grades must be severally added to those of like denomination, during the process. In reducing to higher orders, remainders are synonymous with their respective dividends.

PROOF.

Reduce the resulting numbers to their original denominations.

FEDERAL MONEY, OR MONEY OF THE UNITED STATES.

The denominations are,—

10 mills (*m.*) make 1 cent,.......*c.*
10 cents...............1 dime,......*d.*
10 dimes...............1 dollar,.....$ or *D.*
10 dollars.............1 eagle......*E.*

STANDARD WEIGHT.

The Eagle contains 270 grains of standard gold, 11 parts of which out of 12 are pure gold, the other twelfth alloy, composed of silver and copper.

The Dollar contains 416 grains of standard silver, consisting of 1485 parts of pure silver to 179 of copper.

The Cent contains 208 grains of copper.

Note.—Any given sum in Federal money may be reduced to its lower denominations, by merely annexing one or more ciphers; thus to reduce eagles to dollars, dimes, cents, or mills, annex one, two, or three ciphers respectively. On the other hand, the lower denominations are reduced to higher, by separating one or more right hand figures thus to reduce cents to dollars, separate the two right hand figures for cents, and the rest are dollars.

Also cents may be read as dollars and cents; as dollars, dimes and cents, &c.; thus 26475 cents, may be read 26 eagles 4 dollars 75 cents; 264 dollars 75 cents; 2647 dimes 5 cents; or 26 eagles 4 dollars 7 dimes 5 cents.

EXAMPLES.

1. In 476546 mills how many dimes?
2. Required the number of dollars in 27358 cents.
3. In 468 dollars how many cents?
4. Convert 38 eagles 4 dollars into mills.
5. Required the number of dimes in 276 eagles.
6. Quere the number of eagles in 29648 cents?
7. In 37 E. 5 $ 7 d. 4 cts. how many mills?
8. In 97454 mills how many dollars?
9. Reduce 874 dollars 16 cents to cents.

STERLING OR ENGLISH MONEY.

The denominations are,—

4 farthings (marked *qr.*) make	1 penny, marked *d.*
12 pence..................................	1 shilling,............*s.*
20 shillings................................	1 pound,............£
21 shillings................................	1 guinea.

EXAMPLES.

1. In £376 how many shillings and pence? *Ans.* 7520*s.* 90240*d.*

2. Reduce £56 14*s.* 6*d.* to pence. *Facit* 13614*d.*

3. In £165 13*s.* how many farthings? *Ans.* 159024*qrs.*

4. In 180960*d.* how many pounds? *Ans.* £754.

5. Reduce 36498*qrs.* to pounds. *Facit* £38 0*s.* 4½*d.*

6. In 48 guineas how many pence sterling? *Ans.* 12096*d.*

7. Reduce 14364 pence sterling to guineas. *Facit* 57 *guineas.*

8. Reduce £967 14*s.* 7*d.* to pence. *Result* 232255*d.*

9. In 135764*qrs.* how many pounds? *Ans.* £141 8*s.* 5*d.*

10. Reduce £197 15*s.* 4*d.* to qrs. *Facit* 189856*qrs.*

11. Reduce 878 guineas to pence and these pence to pounds. *Facit* 189856*d.* and £921 18*s.*

12. Reduce 367 guineas to qrs. *Facit* 369936 *qrs.*

TROY WEIGHT.

By this weight, jewels, gold, silver and liquors are weighed.

The denominations are,—

24 grains (*grs.*)	1 pennyweight, *dwt.*[*]
20 pennyweight	1 ounce. *oz.*
12 ounces	1 pound, *lb.*

---

[*] Originally 32 grains of wheat were used to determine the weight of a silver penny, or pennyweight. See note, page 10, Part II.

EXAMPLES.

1. In 27*lb.* 10*oz.* 13*dwts.* of gold, how many grains?
Ans. 160632.
2. In 8*lb.* 0*oz.* 7*dwts.* 2*grs.* how many grains?
Ans. 46250.
3. Reduce 158262*grs.* to pounds.
Facit 27*lb.* 5*oz.* 14*dwts.* 6*grs.*
4. Reduce 376457*grs.* to pounds.
Facit 65*lb.* 4*oz.* 5*dwts.* 17*grs.*
5. In 375*lb.* 10*oz.* 16*grs.* how many grains?
Ans. 2164816*grs.*
6. In 167597*dwts.* how many pounds?
Ans. 698*lb.* 3*oz.* 17*dwts.*
7. In 25*lb.* 9*oz.* 10*dwts.* how many grains?
Ans. 148560*grs.*
8. Reduce 97645745*grs.* to pounds.
Facit 16952*lb.* 4*oz.* 12*dwts.* 17*grs.*

AVOIRDUPOIS WEIGHT.

By this weight are weighed things of a coarse, drossy nature, that are bought and sold by weight, and all metals but silver and gold.

The denominations are,—

16 drams (*dr.*) make . . 1 ounce, . . *oz.*
16 ounces . . . . 1 pound, . . *lb.*
28 pounds . . . . 1 quarter of a *Cwt qr.*
4 quarters, or 112*lb* . . 1 hundred weight, *Cwt.*
20 hundred weight . . . 1 ton, . . . *T.*

The pound avoirdupois weighs 7000*grs.* troy, consequently 144*lb.* avoirdupois are equal to 175*lb.* troy; and 192*oz.* avoirdupois are equal to 175*oz.* troy.

Some things are bought and sold by the dozen, gross, &c. Hence

12 particulars make . . . 1 dozen, . . *doz.*
12 dozen . . . . 1 common gross, *gro.*
12 common gross, or 144*doz.* 1 great gross, *g. gro.*
20 particulars . . . . . . 1 score, . . *sco.*

EXAMPLES.

1. In 15 *tons,* how many hundred weight, quarters and pounds?

Ans. 300*cwt.* 1200*qrs.* 33600*lb.*

2. Reduce 67200*lb.* to tons.

Facit 30 *tons.*

3. In 9*cwt.* 5*lb.* how many ounces? *Ans.* 16208*oz.*

4. Reduce 20571005 *drams* to tons.

Facit 35*t.* 17*cwt.* 1*qr.* 23*lb.* 7*oz.* 13*dr.*

6. One of the stones in the walls of Balbec is said to have weighed 683*t.* 8*cwt.* quere its weight in pounds?

Ans. 1530816*lbs.*

6. In 57*t.* 10*c.* 3*qrs.* 14*lb.* how many drams?

Ans. 32997888*qrs.*

7. Reduce 4768768 drams to tons.

Facit 8*T.* 6*C.* 1*qr.* 8*lb.*

8. Reduce 76*T* 15*C.* to pounds and these again to tons.

Facit 171920*lbs.*

APOTHECARIES' WEIGHT.

By this weight apothecaries mix their medicines.

The denominations are,—

20 grains (*gr.*) make 1 scruple, ℈
3 scruples . . . 1 dram, ʒ
8 drams . . . . 1 ounce, ℥
12 ounces . . . 1 pound, ℔

The pound, ounce, and grain, are the same as in troy weight.—These denominations are used in the composition of medicines. Drugs when purchased in large quantities are weighed by the avoirdupois pound.

EXAMPLES.

1. In 17℔ how many ounces, drams, and scruples?

Ans. 204℥, 1632ʒ, 4896℈

2. In 1332005 *grains,* how many pounds?

Ans. 231℔, 3℥ 5*gr.*

3. In 5℔ of drugs, how many parcels, each 16 drams?

Ans. 30 parcels.

4. Reduce 27℔ 3℥ 2ʒ to grains. *Facit* 157080 *grs.*

5. In 245 parcels of drugs each 10*oz.* 3*drs.* 2 *scrup.* how many pounds? *Ans.* 213*lb.* 6*oz.* 2*drs.* 1*scrup.*

LONG MEASURE.

This measure is used for lengths and distances.

The denominations are,—

3 barley corns (*b.c.*) make	1 inch,	*in.*
12 inches	1 foot,	*ft.*
3 feet	1 yard,	*yd.*
5½ yards	1 rod, pole, or perch.	*P.*
40 poles (or 220 *yds.*)	1 furlong,	*fur.*
8 furlongs (or 1760 *yds.*)	1 mile,	*M.*
3 miles	1 league,	*L.*
60 geographic miles	1 degree,*	*deg.*

360 degrees the circumference of the earth.

Note.—A hand is a measure of 4 inches, and particularly applied to measuring the heights of horses; and the fathom of 6 feet, to the depth of water.

EXAMPLES.

1. How many inches are in 273 *miles*?
Ans. 17297280 *in.*

2. In 34594560*in.* how how many miles?
Ans. 546*m.*

3. Reduce 2*m.* 1*fur.* 8*p.* 3*yds.* 2*in.* into inches.
Facit 136334 *in.*

4. Reduce 2280060 *barley-corns* to miles.
Facit 11*m.* 7*fur.* 38*p.* 2*yds.* 2*ft.*

5. Required the number of revolutions a wheel 18*ft.* 4*in.* will make in running 150 *miles.*
Facit 43200.

6. What distance must a measuring wheel 18*ft.* 4*in.* in circumference, run, to make 86400 turns?
Facit 300*m.*

7. From Philadelphia to New York is 95 miles, how many barleycorns would occupy this distance?
Ans. 18047600.

---

* According to the latest determinations, a degree of the equator is about 69 1-6 statute miles, and that of the meridian, at a mean, 69 1-12.

8. The contemplated rail-road from Philadelphia to Columbia is 83 miles 32 perches in length, what is this distance in feet? *Ans.* 438768.

9. The circumference of the equator being 24896 miles, how many steps of 2 feet 9 inches must a man take in walking round it? *Ans.* 47800300.

CLOTH MEASURE.

By this measure cloths, tapes, &c. are measured.

The denominations are,—

2¼ inches (*in.*) make	1 nail,	*na.*
4 nails	1 quarter of a yard,	*qr.*
4 quarters	1 yard,	*yd.*
3 quarters	1 ell Flemish,	*E. Fl.*
5 quarters	1 ell English or French,	*e.E.e.Fr.*
2½ quarters or 10 nails	1 ell Hamburgh,	*E.H.*

EXAMPLES.

1. In 15*yds.* 3*qr.* 1*na.* how many nails? *Ans.* 253*na.*

2. In 1012 *nails* of cloth, how many yards? *Ans.* 63*yds.* 1*qr.*

3. Reduce 73 ells Flemish to quarters. *Facit* 219*qrs.*

4. How many ells Flemish are in 1752 *nails*? *Ans.* 146 *ells.*

5. How many ells English are in 1408 *nails*? *Ans.* 70 *E.E.* 2*qrs.*

6. In 47656 nails how many ells English? *Ans.* 2382*E. E.* 4*qrs.*

7. Reduce 475 ells English to nails and these again to yards. *Result* 9500*na.* & 593*yds.* 3*qrs.*

8. In 10 bales of cloth, each 10 pieces, and each piece 12 *yds.* how many yards? *Ans.* 1200*yds.*

9. In 408*yds.* 3*qr.* of cloth, how many ells Flemish; also, how many ells English? *Ans.* 545 *E.Fl.* 327 *E.E.*

10. In 4 bales of cloth, each 12 pieces, and each piece 24 *ells English*, how many yards, and *ells Flemish*? *Ans.* 1440 *yds.* 1920 *E.Fl.*

LAND MEASURE.

This measure is used in designating the quantity of lands.

The denominations are,—

9	square feet (*Ft.*) make	1 yard,	*Yd.*
30¼	yards	1 perch,	*P.*
40	perches	1 rood,	*R.*
4	roods	1 acre.	*Ac.*

EXAMPLES.

1. In 24*ac.* 2*r.* 26*p.* how many perches? *Ans.* 3946*p.*

2. Reduce 365*ac.* 3*r.* 13*p.* to perches. *Facit* 58533*p.*

3. In 267464 perches, how many acres? *Ans.* 1671*ac.* 2*r.* 24*p.*

4. Reduce 54*ac.* 3*r.* 24*p.* to perches, and these again to acres. *Facit* 8784*p.*

5. If 2599200*p.* be divided into 25 equal tracts, how many acres will be in each? *Ans.* 649*ac.* 3*r.* 8*p.*

6. Reduce 27645 perches to acres. *Result* 172*a.* 3*r.* 5*p.*

7. In 493476 perches how many acres? *Ans.* 3084*a.* 36*p.*

8. Reduce 275*a* 3*r.* 27*p.* to perches. *Result* 44147.

9. Reduce 179*a.* 3*r.* 16*p.* to perches. *Result* 28776*p.*

LIQUID MEASURE.

This measure is used for beer, cider, wine, &c.

The denominations are,—

4	gills (*gills*) make	1 pint,	*pt.*
2	pints	1 quart,	*qt.*
4	quarts	1 gallon,	*gal.*
63	gallons	1 hogshead,	*hhd.*
2	hogsheads	1 pipe or butt,	*pi.* or *bt.*
2	pipes or 4 hogsheads	1 tun,	*T.*

Note.—By a law of Pennsylvania, 16 gallons make 1 half-barrel; 31½ gallons 1 barrel; 64 gallons 1 double barrel; 84 gallons 1 puncheon; 42 gallons 1 tierce.

EXAMPLES.

1. In 5*hhds.* of wine how many pints?
Ans. 2520*pts.*

2. Reduce 27*hhds.* 16*galls.* to quarts.
Result 6868*qts.*

3. In 36748 *pints* how many hogsheads?
Ans. 72*h.* 57*g.* 2*qts.*

4. Required the number of tuns in 314568*qts.*
Ans. 312*t.* 18*gals.*

5. In 942 *tuns* how many pints?
Ans. 1899072*pts.*

6. In 120*galls.* 2*qts.* 1*pt.* how many pints?
Ans. 765*pts.*

7. In 32*t.* 1*h.* 25*g.* 3*qts.* 1*pt.* how many pints?
Ans. 65223*pts.*

8. In 108864 gills how many hogsheads?
Ans. 54*hhds.*

9. In 72 hhds. how many gills? *Ans.* 145152*gills.*

DRY MEASURE.

This measure is used for grain, fruit, salt, &c.

The denominations are,—

2 pints (*pt*)	make	1 quart,	*qt.*
8 quarts		1 peck,	*p.*
4 pecks		1 bushel,	*bu.*

Note.—In Europe they sometimes use the quarter containing 8 bushels, in designating the quantity of grain; and the chaldron of 36 bushels as a measure of coals.

EXAMPLES.

1. In 5054 *pints* how many bushels?
Ans. 78*bu.* 3*p.* 7*qts.*

2. In 27 *bu.* 3*p.* how many quarts? *Ans.* 888*qts.*

3. How many sacks each to hold 5*bu.* 2*p.* may be filled out of 4 granaries containing 64*bu.* 2*p.* 4*qts.* each?
Ans. 47.

4. Reduce 257*bush.* 2*p.* 3*qts.* into pints. *Ans.* 16496.
5. In 3040 pints how many bushels? *Ans.* 47*b.* 2*p.*
6. Reduce 539 bushels 3 pints to quarts.
Result 17272.

Note.—Though the terms pint and quart are used both in liquid measure and dry, yet the dimensions of the quart are not uniform.—In wine measure, the gallon contains 231 solid inches; in ale measure 282; the bushel 18½ inches in diameter and 8 deep, contains 2150⅖ inches; consequently the quart is in wine measure 57¾ inches; ale measure 70½ inches; dry measure 67⅕ inches.

TIME.

The denominations are,—

60 seconds (*sec.*) make	1 minute,	*M.*
60 minutes	1 hour,	*H.*
24 hours	1 day,	*D.*
7 days	1 week,	*W.*
4 weeks	1 month,	*M.*
13 months, 1 day and 6 hours, or 365 days & 6 hrs.	1 year,	*Y.*

Note. A common year consists of 365 days, and every fourth, called leap year, of 366.*

The year is also divided into twelve calendar months, as follow:—

The fourth, eleventh, ninth, and sixth,
Have thirty days to each affixed;
And ev'ry other thirty-one,
Except the second month alone,
Which has but twenty-eight in fine,
Till leap year gives it twenty-nine.

EXAMPLES.

1. Reduce 37*w.* 5*d.* into minutes. *Facit* 380160*m.*
2. Reduce 24796800 *sec.* to weeks. *Facit* 41*w.*

---

* This is called the Julian account. According to the Gregorian or new style, every centurial (or hundredth) year, when the number of hundreds is not divisible by 4, is a common year. Other years of the Christian era, when their number is divisible by 4, are leap years.

3. How many hours, minutes and seconds are there in a year? *Ans.* 8766*h.* 525960*m.* 31557600 *sec.*

4. From the creation of the world, 4004 years before Christ, to the year 1824 inclusive, how many days have passed? *Ans.* 2128677 *days.*

5. How many seconds are there in 575 Julian years? *Ans.* 18145620000.

6. In 3629124000 seconds how many Julian years? *Ans.* 115*yrs.*

7. In a tropical year of 365 days 5 hours 48 minutes 48 seconds, how many seconds? *Ans.* 31556928.

8. The Chaldean period consists of 223 lunations of 29 days 12 hours 44 minutes 3 seconds, how many seconds are there in this period? *Ans.* 2551443.

MOTION OR CIRCLE MEASURE.

The denominations are,—

60 second*s* (″) make	1 minute,	′
60 minutes,	1 degree,	°
30 degrees,	1 sign,	*s*
12 signs, or 360°.	1 revolution of a planet.	

EXAMPLES.

1. In 4*s.* 20° 15′ 34″ how many seconds? *Ans.* 504934″.

2. How many minutes and seconds in one complete revolution of a planet? *Ans.* 21600′ 1296000″.

The planet Venus at its greatest elongation appears about 166800 seconds from the sun; required the number of degrees? *Ans.* 46° 20′.

4. In 167573 seconds how many degrees? *Ans.* 46° 32′ 53″.

5. In 162000 seconds how many signs? *Ans.* 15*s.*

6. The planet Venus, when its appearance is the most brilliant, has an elongation of 39° 44′, what is this in seconds? *Ans.* 143040″.

PROMISCUOUS QUESTIONS.

1. In $475 how many cents? *Ans.* 47500 *cts.*

2. Reduce $356 15*cts.* to cents. *Facit* 35615 *cts.*

3. Convert 57000*cts.* into eagles. *Facit* 57 *E.*

4. In £25 14*s.* 1*d.* how many pence? *Ans.* 6169*d.*

5. In 11520*d.* how many pounds? *Ans.* £48.

6. In 27*lb.* 6*oz.* of gold how many grains? 158400 *grs.*

7. In 12960 *grs.* of gold how many pounds? *Ans.* 2*lbs.* 3*oz.*

8. In 8 ingots of silver, each weighing 7*lb.* 4*oz.* 17*dwt.* 15*grs.* how many grains? *Ans.* 341304 *grs.*

9. In 14769 ounces (avoirdupois) how many *Cwt.*? *Ans.* 8*Cwt.* 27*lb.* 1*oz.*

10. In 32 bags of hops, each 2*Cwt.* 1*qr.* 14*lb.* how many pounds? *Ans.* 8512*lb.*

11. In 250 pipes of wine how many gallons, and supposing each pint to weigh a pound, what is their weight in tons? *Ans.* 31500*gals.* and 112*T.* 10*cwt.*

12. How many perches are there in 125*ac.* 2*r.* 16*p.* of land? *Ans.* 20096 *p.*

13. A section of land containing 640*ac.* being divided into 25 equal shares; quere the number of perches in each. *Ans.* 4096*p.*

14. How many steps of 2 *ft.* 9*in* each would a person take in walking from Philadelphia to Harrisburg, the distance being 97 *miles?* *Ans.* 186240.

15. How many seconds are there in a Julian year? *Ans.* 31557600*sec.*

16. In a sidereal year of 365 days 6 hours 9 minutes 11 seconds how many seconds? *Ans.* 31558161.

17. In 25 boxes of drugs, each 27℔ 10℥ how many scruples? *Ans.* 200400.

18. Reduce 10*s.* 14° 16′ 25″ into seconds. *Facit* 1131385″.

19. In 3824 pieces each 12½*cts.* how many dollars? *Ans.* 478.

20. In 3562 American dollars how many pounds sterling; the dollar being worth 4*s.* 7*d.* sterling? *Ans.* £816 5*s.* 10*d.*

21. In 81900*d.* sterling how many English guineas? *Ans.* 325*G.*

22. The Julian year contains 365 days 6 hours and the tropical year 365 days 5 hours 48 minutes 48 seconds, how many hours, minutes and seconds are there in the difference between 100 Julian and as many tropical years? *Ans.* 18*hrs.* 40*min.*

23. Required the difference between 235 lunations of 29 days 12 hours 44 minutes 3 seconds, and 19 tropical years of 365 days 5 hours 48 minutes 48 seconds each.

Result 2*hrs.* 4*min.* 33*sec.*

Note. This near approximation of 19 years to 235 lunations gave rise to the lunar or Metonic cycle, a period of 19 years; at the end of which, the new and full moons return nearly to the same day of the month.

◆

COMPOUND ADDITION.

Compound addition is the collection of compound numbers, or numbers of divers denominations, into one sum.

RULE.

Set the numbers of like denominations under each other, so that the lower names shall stand on the right of the higher: Add up those of the lowest denominations, reduce the sum to the next higher name, and the sum thus obtained to the next column, setting down the remainder, if any, under its like denomination, and so proceed.

PROOF.

See methods 1, and 2, in Addition of Integers.

EXAMPLES.

$	*cts.*	$	*cts.*	$	*cts.*
136	26	3579	42	8465	73
573	59	4352	65	1736	98
842	65	2435	23	5260	43
736	57	5374	62	7458	31
569	73	6220	57	1534	26
426	40	5647	34	8263	01
157	34	7564	76	4739	56

£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
364	10	6	527	6	10½	137	16	4½
425	16	9	393	14	3¾	639	6	8¼
336	15	3	426	9	8	523	10	4¾
635	9	5	472	13	1½	635	7	3½
574	3	2	606	5	8½	852	13	6
						678	18	7¾

lb.	*oz.*	*dwts.*	*grs.*
16	10	14	16
23	8	7	20
52	10	18	15
83	1	5	7
76	3	12	3

lb.	*oz.*	*dwts.*	*grs.*
273	1	15	14
342	9	3	5
657	10	13	21
724	9	10	17
652	7	18	23

T.	*C.*	*qrs.*	*lb.*
13	11	2	15
9	13	1	12
7	9	3	24
23	16	1	19
14	12	0	26

T.	*C.*	*qrs.*	*lb.*
36	15	0	23
57	12	3	16
96	7	2	18
52	19	1	25
17	13	2	26

lb.	*oz.*	*drs.*
12	13	14
236	12	15
98	10	12
154	11	9
85	9	6

T.	*C.*	*qrs.*	*lb.*	*oz.*	*drs.*
18	15	2	23	13	15
29	17	3	20	10	12
36	18	1	26	8	13
57	9	3	17	14	11
63	14	0	10	12	9

℔	℥	ʒ	℈	*grs.*
14	8	4	1	15
25	7	6	2	17
34	10	5	1	16
57	9	3	0	10
19	8	5	2	19

m.	*f.*	*p.*
63	5	27
39	1	19
53	7	31
82	3	12
60	7	21

m.	*f.*	*p.*
39	6	15
54	3	20
67	7	29
60	1	24
45	4	19

yd.	*ft.*	*in.*	*b.c.*
13	2	9	1
57	1	7	2
63	0	11	2
86	1	3	2
42	2	5	1

yd.	*qrs.*	*na.*
63	2	1
57	3	2
45	1	1
47	2	2
53	1	3

yd.	*qrs.*	*na.*
34	2	2
65	5	1
79	2	3
45	1	0
34	0	2

EE.	*qrs.*	*na.*
59	3	2
75	4	3
68	2	1
40	1	2
24	0	3

ac.	*r.*	*p.*
256	3	25
436	1	31
573	2	27
637	3	13

ac.	*r.*	*p.*
76	2	15
65	3	26
57	1	19
98	3	27

ac.	*r.*	*p.*
67	3	29
69	1	16
32	2	25
92	1	36

tun.	*hhd.*	*gal.*
56	2	56
93	3	27
387	1	16
439	2	19
624	3	20
43	1	31

hhd.	*gall.*	*qt.*
32	15	3
69	37	2
42	25	1
11	17	3
60	61	1
25	32	2

gall.	*qt.*	*pt.*
76	3	1
189	2	1
364	1	0
64	3	1
352	2	1
68	3	0

bu.	*pc.*	*qts.*
35	3	6
16	2	3
73	1	4
57	3	2
63	1	7
94	3	5

bu.	*pc.*	*qts.*
34	1	3
79	3	5
84	2	7
91	1	4
65	3	5
74	2	2

bu.	*pc.*	*qt.*
156	2	3
78	8	7
99	1	6
47	3	4
74	2	5
36	1	6

hrs.	*min.*	*sec.*
53	16	35
75	23	41
69	10	51
42	18	27
83	17	39
80	55	16

d.	*hrs.*	*min.*	*sec.*
27	15	32	41
63	20	16	31
72	12	13	40
63	11	29	6
87	21	45	50
28	13	14	19

hrs.	*min.*	*sec.*
9	16	35
8	16	41
19	43	29
7	15	57
8	13	47
11	14	13

APPLICATION.

1. In $47 16 *cts.*, $35 16*cts.*, $9 17*cts.*, $25 83*cts*, and $17 65*cts.*, how many dollars and cents?

Ans. $134 97*cts.*

2. If a horse be bought for $125 50*cts.*, a chaise for $147 75*cts.*, and harness for $25 25*cts.*, what is the amount?

Ans. $298 50*cts.*

3. Deposited in bank at sundry times; first, $527 18*cts.*, secondly, $795 25*cts.*, thirdly, $157 28*cts.*, and lastly, $347 75*cts.* for what sum may I draw?

Ans. $1827 46*cts.*

4. If a jeweller buys 5 ingots of silver weighing, 1st, 4*lb.* 10*oz.* 13*dwts.*; 2d, 3*lb.* 4*oz.* 16*dwt.* 19*grs.*; 3d, 5*lb.* 10*oz.* 16*dwts.* 10*grs.*; and each of the others 3*lb.* 10*oz.* 17*dwts.* 18*grs.*; what is the weight of the whole?

Ans. 22*lb.* 0*oz.* 1*dwt.* 17*grs.*

5. In 6*hhds*. tobacco, weighing as follows; No. 1, 13*C*. 2*qrs*. 12*lb*.; No. 2, 15*c*. 3*qr*. 19*lb*.; No. 3, 14*c*. 1*qr*. 23*lb*.; No. 4, 17*C*. 3*qrs*. 10*lb*.; No. 5, 16*C*. 1 *qr*. 18*lb*.; No. 6, 10*C*. 2*qr*. 25*lb*.; how many parcels each 41*lb*.?

Ans. 243.

6. Bought 5 bales of cloth containing No. 1, 42*yds*. 3 *qrs*.; No. 2, 35*yds*. 2*qrs*.; No. 3, 39*yds*. 3*qrs*.; No. 4, 45 *yds*. 3 *qrs*.; No. 5, 27*yds*. 2*qrs*.; how many yards are there in the whole? And admitting each quarter to cost $1 25 *cts*.; what is their value? *Ans*. 191 *yds*. 1*qr*. *Value* $956 25.

7. In one tract of land there are 235*a*. 2*r*. 16*p*. in another 346*a*. 1*r*. 29*p*. in a third, 198*a*. 3*r*. 35*p*. and in each of two others 225*a*. 2*r*. 28*p*.; how many acres in all?

Ans. 1232 *a*. 1*r*. 16*p*.

8. The great bell at Moscow, the largest in the world, weighs 198*T*. 2*C*. 1*qr*.; the bell at Oxford, the largest in England, weighs 7*T*. 11*C*. 3*qr*. 4*lb*.; St. Paul's bell at London, 5*T*. 2*C*. 1*qr*. 22*lb*.; and the Tom of Lincoln, 4*T*. 16*C*. 3*qr*. 18*lb*.; what is the sum of their weights?

Ans. 215*T*. 13*C*. 1*qr*. 16*lb*.

9. If one cistern contains 25*hhds* 27*galls* 3*qts*. a second 37*hhd* 26*gall* 2*qt*. a third 35*hhd* 54*gall* 1*qt*. and a fourth 45*hhd* 15*gall* 3*qt*. what quantity will they jointly contain? *Ans*. 143*hhd* 61*gal* 1*qt*.

COMPOUND SUBTRACTION.

Compound subtraction is the taking of a quantity of several denominations from a greater of like quality.

RULE.

Place the quantities as in compound addition, with the less under the greater; then begin at the right hand, and, if possible, take the under from the upper, and set down the remainder; but when the lower number is greater than the upper, add to the upper as many of that denomination as make a unit of the next greater, then subtract, set down the remainder, carry one to the next term of the subtrahend: and so proceed.

PROOF.

See methods 1st and 2d in subtraction of integers.

EXAMPLES.

$	cts.
365	16
137	25

$	cts.
26	67
8	98

$	cts.
967	53
187	76

$	cts.
436	87
139	69

£	s.	d.
237	10	6
185	14	3

£	s.	d.
47	10	4
23	18	7½

£	s.	d.
367	13	4½
179	18	7¾

lb.	oz.	dwts.	grs.
36	10	13	16
18	11	16	22

lb.	oz.	dwts.	grs.
167	3	10	12
93	9	17	23

lb.	oz.	dwts.	grs.
39	9	16	15
20	10	18	20

T.	C.	qrs.	lb.
369	11	1	15
138	15	3	13

T.	C.	qrs.	lb.
17	14	2	13
9	15	3	27

lb.	oz.	drs.
26	10	11
18	13	15

℔	℥	ʒ	℈	grs.
16	3	4	1	10
9	7	6	2	16

℔	℥	ʒ	℈	grs.
35	9	5	2	13
19	11	6	1	18

℔	℥	ʒ	℈	grs.
56	2	6	1	11
27	3	7	2	19

m.	f.	p.
156	3	14
78	6	16

m.	f.	p.
86	2	13
27	5	34

yds.	ft.	in.	b.c.
379	1	6	2
164	2	11	1

yds.	qrs.	na.
573	2	1
196	3	2

yds.	qrs.	na.
65	1	2
43	2	3

E.E.	qrs.	na.
75	2	2
68	4	3

E.F.	qrs.	na.
43	1	1
27	2	3

a.	*r.*	*p.*
253	1	19
169	2	25

a.	*r.*	*p.*
36	2	25
19	3	36

a.	*r.*	*p.*
16	0	10
7	2	28

hhd.	*gal.*	*qt.*
29	15	2
16	38	3

Hhds.	*gall.*	*qts.*
378	37	3
186	35	1

gall.	*qts.*	*pt.*
59	1	1
27	3	2

bu.	*p.*	*qts.*
54	2	5
23	3	7

bu.	*p.*	*qts*
35	1	2
19	2	5

bu.	*p.*	*qts.*
58	2	3
39	3	6

D.	*hrs.*	*min.*	*sec.*
26	10	16	25
15	16	26	42

D.	*hrs.*	*min.*	*sec.*
27	13	25	41
9	17	34	26

APPLICATION.

1. Borrowed $345 50*cts.* and paid in part $127 75*cts.* how much remains unpaid? *Ans.* $217 75*cts.*

2. Deposited in bank in gold 25 *E.* in silver $75 25*cts.* in notes $326, and drew checks for $239 75*cts.* and $131 84*cts.* what sum is left? *Ans.* $279 66*cts.*

3. A trader failing was indebted to A. $347 57*cts.* to B. $569 84*cts,* to C $875 65cts, and to D $956 84cts. When this happened, he had in cash $137 84*cts.*; goods worth $785 76*cts.*; household furniture, worth $374 26*cts.*; and in book debts $458 67*cts.*; if these were given up to his creditors, how much did they lose by him? *Ans.* $993 37*cts.*

4. What is the difference between £347 10*s.* 7*d.* and £239 19*s.* 9½*d.*? *Ans.* £107 10*s.* 9½*d.*

5. The weight of three bars of silver taken together is 14 *lb.*; of which the first weighs 4*lb.* 3*oz.* 15 *dwts.*, and the second, 5*lb.* 7*oz.* 13*dwts.* 20*grs.*; required the weight of the third. *Facit* 4*lb.* 0*oz.* 11*dwts.* 4*grs.*

6. What difference is there between the great bell of Oxford and the Tom of Lincoln, the former weighing 7*T.* 11*C.* 3*qrs.* 4*lb.*; and the latter 4*T.* 16*C.* 3*qrs.* 18*lb*? *Ans.* 2*T.* 14*C.* 3*qrs.* 14*lb.*

7. A person purposing to travel from New-York to Boston, a distance of 250 *miles,* in five days, travels 46*M.* 2*fur.* 25*po.* the first day, 48*M.* 3*fur.* 16*po.* the second,

53*M.* 4*fur.* 25*po.* the third, and 47*M.* 5*fur.* 37*po.* on the fourth; how far must he travel the last day to complete the journey? *Ans.* 53*M.* 7*fur* .17*po.*

8. From a tract of land containing 435*ac.* 2*r.* 25*po.* a part was cut off, containing 129*ac.* 3*r.* 30*po.*; how much was left? *Ans.* 305*ac.* 2*r.* 35*po.*

9. Sir Isaac Newton, the father of true physical astronomy, was born 1642, 12th month, (December) 25th, and died 1727, 3d month, (March) 10th; required his age?"*

Ans. 85*yrs.* 2*mo.* 16*days.*

10. A man was born on the 13th of the 6th month (June) 1740, (old style) and died on the first day of the 3d month 1808, (new style) how old was he?

Ans. 67*yrs.* 8*mo.* 7*day.*

11. If a ship sail from St. Petersburg, in Russia, on the 14th of 9th month, 1820, and arrive at Philadelphia on the 13th of 1st month, 1821; what is the time of the passage?

Hns. 3*mo.* 17*days.*

12. If from a cistern containing 68*hhds.* 16*galls.* 1*qt.* there be drawn at one time, 15*hhds.* 27*galls.* 2*qts.*, at another, 27*hhds.* 19*galls.* 2*qts.*, and at a third 16*hhds.* 10*galls* 3*qts.*; how much remains?

Ans. 9*hhds.* 21*galls.* 2*qts.*

COMPOUND MULTIPLICATION.

Compound multiplication is a rule by which we obtain the amount of a quantity consisting of divers denominations when repeated a given number of times.

---

* The interval between two given dates may be found, with peculiar facility, when the plain style is used; thus arrange the terms as in rule, and if the days in the subtrahend exceed those in the minuend, add, to the latter, the number of days in the month of the prior date, then subtract and carry as in other cases. In the above example, 31days are added to the 10, because the 12th month contains 31 days. When one date is in the old style, and the other in the new, the interval may be found as before, if we first reduce the old style to the new, by adding 11 days to the given date, for any time between the years 1700 and 1800, or 12 days for any time between 1800 and 1900. In Russia the old style is still in use.

CASE 1.

When the multiplier does not exceed 12.

RULE.

Multiply the several denominations in succession, beginning with the lowest; reduce the results and set down and carry as in addition.

PROOF.

See method second, multiplication of integers.

EXAMPLES.

$	cts.
27	37
	4*

$	cts.
256	18
	7

$	cts.
925	63
	9

$	cts.
239	26
	12

£	s.	d.
14	15	6
		3

£	s.	d.
27	16	9
		6

lb.	oz.	dwt.	gr.
15	6	14	20
			8

lb.	oz.	dwt.	gr.
39	10	12	16
			5

C.	qr.	lb.
56	2	23
		11

C.	qr.	lb.	oz.	dr.
15	3	16	10	12
				9

lb.	℥	ʒ	℈	gr.
19	10	6	1	15
				11

lb.	℥	ʒ	℈	gr.
26	11	5	2	16
				8

m.	f.	p.
36	7	29
		12

yd.	ft.	in.
23	2	10
		9

yd.	qr.	na.
29	3	2
		11

yd.	qr.	na.
65	2	3
		7

---

*The examples in federal money are wrought like those in common integers. Consequently all the contractions and methods of proof given in multiplication of integers are equally applicable to the examples in federal money.

CASE 2.

When the multiplier is the exact product of two factors in the multiplication table.

RULE.

Multiply by one of those factors, and that product by the other.

PROOF.

Change the order of the factors.

1. Multiply 132*lb.* 10*oz.* 15*dwt.* 16*gr.* by 36
   Result 4784*lb.* 4*oz.* 4*dwt.*
2. ——— 569*lb.* 6*oz.* 11*dwt.* 20 *gr.* by 54
   Res. 30755*lb.* 7*oz.* 19*dwt.*
3. ——— 12*T.* 10*C.* 1*qr.* 15*lb.* by 63
   Res. 788*t* 14*c* 21*lb.*
4. ——— 29*hhd.* 27*gall.* 3*qt.* 1*pt.* by 72
   Res. 2119*hhd.* 54*gal.*
5. ——— 15*bu.* 3*p.* 6*qt.* by 48
   Res. 765*bu.*
6. ——— 27*yd.* 3*qr.* 1*na.* by 56
   Res. 1557*yds.* 2*qrs.*
7. ——— 35*ac.* 2*r.* 16*p.* by 132
   Res. 4699*a* 32*p.*
8. ——— 16*d.* 10*hr.* 26*min.* 19*sec.* by 121
   *Res.*1988*d.* 15*hrs.* 4*m.* 19*sec.*

Note. When the multiplier is the continued product of three or more factors each not more than 12, multiply as before, using all those factors in succession.

When the multiplier is nearly equal to the product of two or more factors in the table, multiply by those factors as before; then multiply the given multiplicand by the difference between the given multiplier and the product of those factors, and add or subtract the result according as this product is too small or too great.

EXAMPLES.

Multiply 26*ac.* 1*r.* 15*p.* by 140

	ac.	*r.*	*p.*		*ac.*	*r.*	*p.*		*ac.*	*r.*	*p.*
Solution.	26	1	15	or	26	1	15×8	or	26	1	15×4
			4				12				12
	105	1	20		316	0	20		316	0	20
			5				11				12
	526	3	20		3477	1	20		3793	2	0
			7		210	3	0+		105	1	20—
	3688	0	20		3688	0	20		3688	0	20

2. Multiply 365*days* 5*hrs.* 48*m.* 51*sec.* by 165
Result 60264*d.* 23*hrs.* 20*m.* 15*sec.*

3. —— 23*yds.* 2*qrs.* 3*na.* by 225
Res. 5329*yds.* 2*qrs.* 3*na.*

4. —— 3*T.* 14*C.* 1*qr.* 15*lb.* by 76
Res. 282*T.* 13*C.* 0*qr.* 20*lb.*

5. —— 27*bu.* 2*p.* 6*qts.* by 378
Res. 10465*bu.* 3*p.* 4*qts.*

CASE 3.

When the multiplier is large and such that the preceding rules cannot be conveniently applied.

RULE.

Multiply continually by as many tens less one as there are figures in the multiplier; then beginning at the left hand of the multiplier, multiply the last product by the first figure; the next to the last by the second; and so on, observing that the given multiplicand must be multiplied by the units of the multiplier, the sum of these products is the number required.

EXAMPLES.

1. Multiply 15*C.* 1*qr.* 15*lb.* by 3574
Result 54982*C.* 0*qr.* 18*lb.*

2. Multiply *26lb* 10*oz*. 15*dwts*. 18*grs*. by 5673
Res. 152597*lb* 9*oz* 9*dwts*

3. ——— 16*yds* 3*qrs* 3*na* by 2769
Res. 46899*yds* 3*qrs* 3*na*

4. ——— 12*ac* 2*r* 25*p* by 3947
Res. 49954*ac* 0*r* 35*p*

5. ——— 42*hhds* 16*gall* 3*qts* by 5694
Res. 240661*hhds* 55*gals* 2*qts*

6. ——— 25*bu* 2*p* 5*qts* by 6237
Res. 160018*bu* 0*p* 1*qt*

7. ——— 164*d* 15*hrs* 10*m* 35*sec* by 7423
Res. 1222065*d* 22*h* 20*min* 5*sec*

APPLICATION.

1. What is the weight of 47*hhds* of tobacco each weighing 6*C* 2*qrs* 17*lb* ? *Ans*. 312*C* 2*qrs* 15*lb*

2. In 45 ingots of silver, each 3*lb* 7*oz* 14*dwt* 13*gr* how many pounds ? *An*. 163*lb* 11*oz* 14*dwts* 9*gr*

3. In 39 casks of sugar, each 2*C* 1*qr* 13*lb* how many *C qr and lb ?* *An*. 92*C* 1*qr* 3*lb*

4. What is the weight of 9 boxes of drugs each 16℔ 5℥ 6ʒ 2℈ 15grs ? *An*. 148℔ 4℥ 6ʒ 15*gr*.

5. What is the content of a plantation, consisting of nine fields, each containing 10*ac* 2*r* 16*p ?* *An*. 95*ac* 1*r* 24*p*

6. How many days are there in 15 tropical years of 365 *days* 5*hr* 48*m* 51*sec* each ? *An*. 5478*d* 15*hr* 12*min*

7. In 25 bales of cloth each containing 17 pieces, and each piece 27*yd* 2*qr* 3*na* how many yards ?
An. 11767 *yd* 0*qr* 3*na*

8. In 13 sacks each containing 3*bu* 2*p* 1*qt* how many bushels ? *An*. 52*bu* 3*p* 7*qt*

9. The planet Mercury moves through 4° 5′ 32″ of the zodiac in a day, how far does it move in a month of 30 days?
Ans. 4*s* 2° 46′

10. The moon's node moves backward 19° 19′ 43″ in a year, how far then does it move in 19 years ?
An. 12*s* 7° 14′ 37″.

11. The planet Jupiter describes one sign of the zodiac in 360 days 21 hours 15 min. in what time does it move through the twelve signs ? *Ans*. 4330*d* 15*h*.

12. In 357 hogsheads of sugar, each 15cwt. 3qrs. 17lb., how many cwt.? *Ans.* 5676cwt. 3qrs. 21lb.

13. If a barrel of flour weighs 1cwt. 3qrs. 10lb., what is the burden of a ship which carries 4357 barrels?
Ans. 400T. 3cwt. 3qrs. 2lb.

COMPOUND DIVISION.

Compound division is the converse of compound multiplication, being a rule for dividing a given quantity, consisting of divers denominations, into a proposed number of equal parts. Or for discovering how often a given compound number is contained in another given compound number of the same kind.

GENERAL RULE.

Arrange the terms as in the preceding rules, then divide the highest denomination by the given divisor, set down the quotient, and reduce the remainder to the next lower denomination, adding the corresponding term of the dividend as in reduction, divide as before, and so proceed.*

EXAMPLES.

£ s. d.
3)16 15 6

lb. oz. dwts. grs.
5)27 10 16 6

lb. oz. dwts.
9)96 4 10

T. C. qrs. lb.
4)19 13 1 26

lb. oz. drs.
7)56 10 12

℔ ℥ ʒ ℈ *grs.*
8)397 8 5 1 16

m. f. p.
5)76 5 36

yd. ft. in.
9)256 2 10

yd. ft. in. bc.
7)265 1 9 2

---

* This rule is applicable to the case in which the divisor is an abstract number, the quotient being then always synonymous with the dividend from which it was derived. When the divisor is a compouud number, reduce it, as well as the dividend, to the lowest denomination contained in either, and divide as in integers. The quotient, in this case, is an abstract number.

	yd.	*qrs.*	*na.*		*yd.*	*qrs.*	*na.*		*ac.*	*r.*	*p.*
8)	567	3	2	10)	975	2	3	9)	517	2	35

When the divisor exceeds 12 and is the product of two or more factors in the multiplication table, proceed as directed in the second method of contractions in Division of Integers. In other cases, with large divisors, use the common method of long division, reducing remainders according to the general rule.

1. Divide £233 10*s* 9*d* by 32
Result £7 5*s* 11½*d*.+
2. ——— 959*lb* 8*oz* 14*dwts* 10*grs* by 56
Res. 17*lb* 1*oz* 13*dwt* 2*gr*+
3. ——— 327*lb* 5*oz* 15*dwts* 20*grs* by 63
Res. 5*lb* 2*oz* 7*dwt* 13*gr*+
4. ——— 536*T* 13*C* 2*qrs* 25*lb* by 45
Res. 11*T* 18 *C* 2*qrs* 3*lb*.+
5. ——— 236*m* 7*f* 28*p* by 36
Res. 6*m* 4*f* 26*p*.+
6. ——— 596*yds* 3*qrs* 2*na* by 48
Res. 12*yds* 1*qr* 2*na*.+
7. ——— 873*bu* 3*p* 5*qt* by 125
Res. 6*bu* 3*p* 7*qts*.+
8. ——— 1027*ac* 2*r* 30*p* by 135
Res. 7*ac* 2*r* 18*p*.
9. ——— 1565*d* 16*hrs* 27*m*. by 354
Res. 4*da* 10*h* 8*min*.+
10. ——— 5769*hhd* 57*gall* 3*qt* by 432
Res. 13*hhd* 22*gall* 1*qt*+

APPLICATION.

1. If 26*yds*. of cloth be sold for $32 50*cts*, what is the price of a yard? *Ans*. $1 25*cts*.

2. Bought 25 yards of cloth for £38 2*s* 6*d*; what was the price of a yard? *Anz*. £1 10*s* 6*d*.

3. If 42*cwt* of flour be packed in 24*bbl*. required the quantity in each.
Ans. 1*C*. 3*qr*.

4. If an ingot of silver weighing 17*lb* 7*oz* 6*dwt* 6*gr* be coined into $245, what is the weight of each, supposing no loss of metal during the process? *Ans*. 17*dwt* 6*gr*.

5. If 8*lb* 5*oz* 11*dwt* 18*gr* of gold be coined into 387 English guineas; what is the weight of each?

Ans 5*dwt* 6*gr*.

6. Imported 135*hhd* of sugar weighing in all 1704*C*. 1*qr* 14*lb*; what was the weight of each?

Ans. 12*C* 2*qr* 14*lb*.

7. The moon revolves through 12 signs of the zodiac in 27*d* 7*hr* 42*min* 48*sec*; in what time does it describe one sign?

Ans. 2*d* 6*h* 38*m* 34*s*.

8. If 57℔ 7℥ 7ʒ 12*gr* of drugs be put into 17 boxes; how much is contained in each?

Ans. 3℔ 4℥ 5ʒ 1℈ 16*gr*.

THE SINGLE RULE OF THREE, OR SIMPLE PROPORTION.

When four numbers are such that the first contains the second, or some aliquot part of the second, as often as the third contains the fourth, or the same part of the fourth, those numbers are said to be proportional; or the first is to the second as the third is to the fourth.*

When the proportionals are applicate numbers, the first must be of the same kind with the second, and the third with the fourth.

The Single Rule of Three is that by which three numbers or terms being given, a fourth proportional is found.

RULE.

Write that number for the third term which is of the same kind with the answer or number sought. Then, consider, from the nature of the question, whether the required answer must be greater or less than this third term; if *greater*, write the greater of the other given numbers for the second term, and the less for the first; but if the required answer must be *less* than the third term, set down the less of the other two for the second, and the greater for the first.

---

* The first and third of these proportional numbers or terms are called antecedents, (or leading terms,) the second and fourth consequents. (or following terms.)

If the first and second terms be of different denominations, reduce both to the lowest in either; and the third to its lowest given denomination. Then,

Multiply the second and third terms together, and divide the product by the first, the quotient will be the fourth term, or answer sought, of the same denomination as that to which the third was reduced.

Note.—The first and second terms, when reduced to the same denomination, may be always considered as abstract numbers, and therefore remainders, as well as quotients, are synonymous with their respective dividends.

PROOF.

Assume the answer found, as one of the given terms, with which, and any two terms given, find the remaining one by the above rule, or when practicable, solve the question by one of the contracted methods hereafter given.

EXAMPLES.

1. If *5lb* of butter cost 75*ct*; how much will 13*lb* cost?

Here the term of the same kind as the required answer is 75*cts.* which we set down as the 3d term. Then the price of 13*lb* is evidently greater than the price of 5*lb*; that is, the required answer is greater than the 3d term, we therefore set down 13*lb* for the 2d term and 5*lb* for the 1st, thus:

lb. *lb.* *cts.*
As 5 : 13 :: 75
13
5)975
Ans. $1 95*cts.*

cts. *cts.* *lb.*
Proof. As 75 : 195 :: 5
5
75)975(13*lb.*
75
225
225

2. If a footman perform a journey in 21 days, when the days are 15 hours in length, in how many days of 9 hours can he perform the same journey?

Here the term similar to the required answer is 21*days*, which is consequently the third term: Then, since the footman will not travel so far in a day of 9*hrs.* as in one of 15 *hrs.*, the required answer must be greater than the 3rd term; the 15*hrs.* must therefore be the 2nd term and 9*hrs.* the 1st, thus:

	hrs.		*hrs.*		*days*			*hrs.*		*hrs.*		*days*
As	9	:	15	::	21	*Proof.*	As	15	:	9	::	35
					15							9
					9)315							15)315
			Ans.		35 *days.*						*days.*	21

3. If 7 bushels of wheat cost $6 65*cts.* what will 15*bu.* cost at the same rate? *Ans.* $14 25*cts.*

4. What is the price of 1*C.* 2*qrs.* 14*lb.* of sugar at the rate of 25*cts.* for 2*lb*? *Ans.* $22 75*cts*

5. If 32 yards of cloth cost $81 92*cts.*; what is the price of one yard? *Ans.* $2 56*cts.*

6. What is the value of 1*cwt.* of sugar at 8*cts.* per pound? *Ans.* $8 96*cts.*

7. If 1*cwt* of iron be worth $4 25*cts.*; what is the value of 33*C* 1*qr* 22*lb*? *Ans.* $142 14*cts.*

8. A bankrupt compounds with his creditors at 65*cts.* in the dollar; what is the dividend on $1256 at that rate? *Ans.* $816 40*cts.*

9. If 3*cwt.* 1*qr.* of iron cost $18 20cts., what will be the price of 25*cwt.* 2*qr.* 14*lb*? *Ans.* $143 50cts.

10. If a firkin of butter, weighing 76lb., cost $9 50cts., what is the price of 374lb? *Ans.* $46 75cts.

11. What do 275 tons of hay cost at the rate of $14 25cts. per ton? *Ans.* $3918 75cts.

12. What is the value of 9lb. 7oz. 15dwts. of silver, at the rate of $1 29cts. per ounce? *Ans.* $149 32cts.

13. What is the value of 15lb. 10oz. 6dwts. 6grs. of pure gold, 2475grs. being worth $100? *Ans.* $3690 90cts. 9m.

Note. There are various expedients, by which, in particular cases, the operations may be considerably shortened.

1. When the second is a multiple or an aliquot part of the first, the fourth is the same multiple or part of the third.

2. When the first, and either of the other given terms, have a common measure, divide them by it, and use the quotients instead of the given numbers.

3. When an aliquot part of the first added to, or subtracted from the first, will produce the second, a like part of the third added to or subtracted from the third will produce the fourth.

14. If 8 yards of cloth cost $20 what will 69*yds.* come to?

$$\begin{array}{lcccccc} & & \textit{yds} & & \textit{yds} & & \$ \\ \text{By note 1} & \text{As} & \cancel{8} & : & \cancel{96} & :: & 20 \\ & & & & 12 & & 12 \\ \cline{7-7} & & & & & \textit{Ans.} & \$240 \\ \cline{7-7} \end{array}$$

15. What do 9*yds* of cambric cost at the rate of $68 40*cts* for 72*yds*?

$$\begin{array}{lcccccccc} & \textit{yds} & & \textit{yds} & & \$ & \textit{cts} & \\ \text{As} & \cancel{72} & : & \cancel{9} & :: & \cancel{68} & \cancel{40} & \\ & \cancel{8} & & & & 8 & .55 & \textit{Ans.} \\ \cline{6-7} \end{array}$$

16. If 24*bu* of wheat cost $27: how much will 60*bu* cost?

$$\begin{array}{lcccccc} & \textit{bu} & & \textit{bu} & & \$ & \\ \text{As} & \cancel{24} & : & \cancel{60} & :: & 27 & \\ & 2 & & 5 & & 5 & \\ \cline{6-6} & & & & & 2)135 & \\ \cline{6-6} & & & & & \$67\ 50\textit{cts} & \textit{Ans.} \\ \cline{6-6} \end{array}$$

17. If a footman perform a journey in 3 days when the days are 16 hours long; how many days will he require, of 12 hours long, to perform the same journey?

$$\begin{array}{lrcrcr}
 & hrs & & hrs & & d \\
\text{As} & 12 & : & 16 & :: & 3 \\
\frac{1}{3}\text{ of} & =4 & & & & 1 \\
 & \text{—} & & & & \text{—} \\
 & 16 & & & & 4 \quad \textit{Ans.}
\end{array}$$

18. If 25*bu* of corn cost $13 75*cts*; what is the value of 135*bu*? *Ans.* $74 25*cts.*

19. What is the value of 576*C* 2*qrs* of iron at the rate of $89 60*cts* per ton? *Ans.* $2582 72*cts.*

20. What will 240*bu* of wheat amount to at the rate of $7 50*cts* per 5 bushels? *Ans.* $360.

21. A person failing in trade, is indebted $25460; and pays 45 cents in the dollar; how much do the creditors lose by him? *Ans.* 14003.

22. If 20 men can accomplish a piece of work in 15 days, in what time will 25 men perform it? *Ans.* 12 *days.*

23. The great wall between China and Tartary is said to have occupied one third of the able-bodied men in the empire 5*yrs* in its erection, now supposing 2000000 of men to have been thus occupied, how many would have been required to finish the same stupendous work in 25*yrs*? *Ans.* 400000.

24. According to the conclusions of modern chymists 85 ounces of oxygen and 15 ounces of hydrogen burnt together form 100 ounces of water, how many ounces of hydrogen are there in a hogshead of water, each gallon weighing 133*oz?* *Ans.* 1256*oz.*

25. If a staff, 4 feet in length, perpendicularly erected, cast a shadow on level ground, 5*ft* 6*in*; what is the height of a steeple whose shadow, at the same time, measures 220 *feet?* *Ans.* 160*ft.*

26. The parallel of latitude passing through Philadelphia, is about 19150 miles, and the earth performs one rotation in 23*hrs* 56*m* 4*sec;* how far then are the Philadelphians carried in an hour by this motion? *Ans.* 800*m.* 0*f.* 32*p.*

27. The flash of a gun was observed 38 seconds before hearing the report; required the distance, sound being supposed* to move at the rate of 1142*ft.* per second? *Ans.* 43396*ft.*

---

* This was Newton's determination. According to the experiments of Dr. Moll and others, made in Holland during the summer of

28. The sentinels' cry, "all's well," at the fort of Gibralter, is said to have been heard, under favourable circumstances, on the African shore at the distance of 12 miles; how long was the sound in moving this distance?
Ans. 55*sec.*+

29. A person in health has about 75 pulsations or beats of the artery in a minute: now a gun being fired on one side of a river, an observer directly opposite counts nine pulsations at his wrist between seeing the flash and hearing the report; what was the breadth of the river?
Ans. 1*m.* 4*f.* 100*yds.* 2*ft.*

30. If 7*C.* 1*qr.* of sugar cost $87 what will be the price of 43*C.* 3*qrs.* 14*lb.*? *Ans.* $526 50*cts.*

31. A person having an income of $2124 25*cts.* a year, saves $500 annually ; how much does he spend in a week?
Ans. $31 15*cts.*

32. If a man takes 5 steps in walking 4 yards, how many steps will he take in walking from Philadelphia to Lancaster, the distance being 64 *miles?* *Ans.* 140800 *steps.*

33. How many men must be employed to finish a piece of work in 15 days, which 24 men can perform in 20 days?
Ans. 32 *men.*

34. The air of our atmosphere consists of nitrogen gas, and oxygen gas, chymically combined in the proportion of 77 parts, by weight, of the former, to 23 of the latter ; how many pounds of oxygen gas are contained in 6*C.* 1*qr.* of atmospheric air? *Ans.* 161*lbs.*

45. If the daily expense of the United States senate be 324 dollars, and they sit 5 hours in the day, how much does a speech in that assembly, of 3 hours in length, cost the public? *Ans.* $194 40*cts.*

36. The yearly income of the president of the United States is $25000, how much is that per hour, the year consisting of 365 days 6 hours? *Ans.* $2 85*cts.*

37. The quantity of gold brought from Ophir by Solomon's fleet at a voyage, is stated in Scripture at 420 talents, how many American eagles would be coined out

---

1822, the velocity of sound in perfectly dry air, at the temperature of 32° Fah. ther., the barometer being at 30in., was 1089 3-5 feet, of this sum, the eagle containing 247½grs. of gold, and the talent being considered to contain 657000grs.?

Ans. 1114909E.+

38. The public debt of the United States at the close of 1826, was about $74000000, query the weight avoirdupois of this sum in silver dollars, each weighing 416grs. troy? *Ans.* 1963T. 5C. 1qr. 2lb.

39. The chambers of Solomon's temple were overlaid with gold, amounting to 600 talents; what was the weight avoirdupois of the gold thus employed, reckoning the talent as in the 37th example?

Ans. 25T. 2C. 3qrs. 6lb.

40. If a ton of anthracite coal cost $8 25cts., what is that per bushel of 80lbs.? *Ans.* 29cts.

THE DOUBLE RULE OF THREE, OR COMPOUND PROPORTION.

When to three given numbers a fourth proportional is required to be found; and to two other given numbers and this fourth term, a second fourth proportional, &c. the rule by which such problems are solved, is called the double rule of three, or compound proportion.*

RULE.

Set down the term which is of the same kind with the required answer in the third place; with this third term and each pair of similar terms complete a stating in the single rule as already taught; then, having reduced the similar terms to like denominations, and the third to its lowest given denomination, multiply the numbers in the second and third places continually together for a dividend, and those in the first for a divisor, divide the former by the latter, and the quotient will be the answer required, of the same denomination as the third term.

---

* When more than two fourth proportionals are required, the term double rule of three appears to be improper; no inconvenience, however, results from its use, as one general method is applicable to all the cases.

PROOF.

Work by two or more statings in the single rule. Or, Assume the answer as given, and compute one of the given terms.

Note. The second method of contraction given in the preceding rule is frequently applicable in this. The principles of the first and third are also applicable, but their applications, especially that of the third, is sometimes too difficult for beginners.

EXAMPLES.

1. If 225 bushels of oats be eaten by 20 horses in 30 days, how many bushels will suffice 50 horses 16 days?

In this example, the term similar to the number sought is evidently 225 bushels; which is therefore set in the third place: then putting the question on the number of horses, the fourth term must be more than the third, for 50 horses will manifestly require more, in a given time, than 20 horses; hence 50 must be the second and 20 the first term. Again, putting the question on the number of days; since a given number of horses will eat less in 16 days than in 30, the fourth term here is less than the third; whence 16 is the second term and 20 the first. Thus.

$$\left.\begin{array}{ccc} \textit{horses} & & \textit{horses} \\ 20 & : & 50 \\ \textit{days} & & \textit{days.} \\ 30 & : & 16 \\ \hline 6|00 & & 800 \end{array}\right\} :: \begin{array}{r} \textit{bu.} \\ 225 \\ 800 \\ \hline 6)1800|00 \\ \hline \textit{Ans.}\ 300 \end{array}$$

$$\text{Proof as}\quad \begin{array}{ccccr} \textit{horses} & & \textit{horses} & & \textit{bu.} \\ \cancel{20} & : & \cancel{50} & :: & 225 \\ 2 & & 5 & & 5 \\ \hline & & & & 2)1125 \end{array}$$

$$\text{As}\quad \begin{array}{ccccr} \textit{days} & & \textit{days} & & \\ \cancel{30} & : & \cancel{16} & :: & 562\frac{1}{2} \\ 15 & & 8 & & 8 \\ \hline & & & & 15)4500(300\ \textit{Ans.} \end{array}$$

Another proof is furnished by the solution of the following question.

2. If 20 horses in 30 days eat 225 bushels of oats; in how many days will 50 horses eat 300 bushels?

$$\begin{array}{ccc} \textit{horses} & & \textit{horses} \\ \end{array}$$
$$\left.\begin{array}{ccc} \cancel{50} & : & \cancel{20} \\ \cancel{5} & & 2 \\ \textit{bu.} & & \textit{bu.} \\ \cancel{225} & : & 300 \\ \cancel{3} & : & 4 \end{array}\right\} :: \begin{array}{c} \textit{days} \\ \cancel{30} \\ \cancel{6} \\ 2 \\ 4 \\ \hline 8 \\ 2 \\ \hline \end{array}$$

Ans. 16 *days*, a given number in the first example.

3. If a family of 8 persons expend $360 in 9 months, how much will serve a family of 18 persons 12 months? *Ans.* $1080.

4. If the wages of 4 men for 7 days be $8 68*cts.* how how many days must 14 men work for $43 40*cts?* *Ans.* 10 *days.*

5. If 3 men in 4 days eat 15*lb.* of bread, how much will suffice 5 men for 12 days? *Ans.* 75*lb.*

6. If $250 in 12 months gain $12 50*cts.* interest, how much will $300 gain in 15 monshs? *Ans.* $18 75*cts.*

7. If the carriage of 8*cwt.* 128 miles, cost $12 80*cts.* how far will 12*cwt.* be carried for $24? *Ans.* 160*m.*

8. If 20*cwt.* be carried 50 miles for $15, how much will 45*cwt.* cost to be conveyed 80 miles? *Ans.* $54.

9. If 12 men in 6 days mow 80 acres; in how many days will 25 men mow 250 acres? *Ans.* 9 *days.*

10. If 5 men make 300 pairs of shoes in 40 days; how many men will make 900 pairs in 60 days? *Ans.* 60*m.*

11. If 144 men can build a wall 32 feet high in 8 days, in how many days can 63 men build a wall 28 feet high of the same length? *Ans.* 16 *days.*

12. If a footman, when the days are 14 hours long, can travel 276 miles in 16 days; in how many days can he travel 828 miles, when the days are but 12 hours long? *Ans.* 56 *days.*

13. If 200*lb.* be carried 40 miles for $1 20*cts.* how far may 20200*lb.* be carried for $60 60*cts?* *Ans.* 20*miles.*

14. If 20 oxen in six weeks eat 30 acres of grass, how long will 24 acres suffice 12 oxen? *Ans.* 8 weeks.

15. If $300 in 18 months gain $27 interest, in what time will $750 gain $97 50cts.? *Ans.* 26 months.

16. If 20 men can accomplish a piece of work in 30 days of 12 hours each, how many men must be employed to effect it in 25 days of 9 hours each? *Ans.* 32 men.

17. If a man walk 552 miles in 16 days of 14 hours each, how far will he walk in 42 days of 12 hours each? *Ans.* 1242 miles.

18. If 15 men in 20 days earn $225, how long will 25 men require to earn $600? *Ans.* 32 days.

19. If 15 men eat $5 40 *cts.* worth of bread in 6 days, when wheat is sold at $1 50*cts.* per bushel; how many days will 30 men require to eat $36 worth of bread, when wheat is sold $1 per bushel? *Ans.* 30 *days.*

20. If 16 compositors set 150 pages types, each page containing 48 lines, and each line 50 letters, in 3 days of 10 hours each; how many compositors will be required to set 500 pages of 72 lines each, and 45 letters in a line, in 6 days of 8 hours each? *Ans.* 45.

PRACTICE.

Practice is a compendious method of ascertaining the value of a given number of articles, as pounds, yards, &c. from the given price of one. It is so called from its frequent application to practical purposes.

The following tables of aliquot parts will assist the student in making calculations according to this method.

cts.			*lb.*			*r.*	*per.*		
50	= ½	of a dollar.	56	= ½	of an cwt.	2	0	= ½	of an acre.
25	= ¼		28	= ¼		1	0	= ¼	
20	= ⅕		16	= ⅐			32	= ⅕	
12½	= ⅛		14	= ⅛			20	= ⅛	
10	= ⅒		8	= 1/14			16	= ⅒	
6¼	= 1/16		7	= 1/16			10	= 1/16	
5	= 1/20						8	= 1/20	

CASE 1.*

When the given price is less than a dollar.

RULE.

Take such part or parts of the given number as the price is of a dollar, for the answer in dollars.

Note.—When the complement of the given price or what it wants of one dollar, is the aliquot part of a dollar, the required answer is conveniently obtained by subtracting from the given number, reckoned as dollars, such part thereof as the complement is of a dollar.

EXAMPLES.

4769 at 35*cts.*

25	$\frac{1}{4}$	4769
10	$\frac{1}{10}$	1192 25
		476 90
		$1669 15

56742 at 87½*cts.*

12½	$\frac{1}{8}$	56742
		7092 75
		$49649 25

			$	cts.
3.	3674 at 50*cts.*	*Facit*	1837	00
4.	2578 at 25*cts.*	—	644	50
5.	9876 at 37½*cts.*	—	3703	50
6.	5764 at 62½*cts.*	—	3602	50
7.	96575 at 12½*cts.*	—	12071	87½
8.	35694 at 31¼*cts.*	—	11154	37½
9.	2646 at 95*cts.*	—	2513	70

CASE 2.

When the given price of an integer consists of dollars and cents.

RULE.

Multiply by the dollars and take parts for the cents as before.

---

* In this case and the following, if the parts cannot be conveniently taken, multiplication may be used in preference to the above method.

EXAMPLES.

25|¼|2567 at $3 25*cts.*	25|¼|576 at $8 75*cts.*
3	9
7701	5184
641 75	144
$8342 75	$5040

		$	cts.		$	cts.
3.	3247 at	4	12½	*Facit*	13393	87½
4.	2798 at	5	56¼	—	15563	87½
5.	5346 at	3	37½	—	18042	75
6.	4985 at	8	75	—	43618	75
7.	2382 at	1	31¼	—	3126	37½
8.	5936 at	3	87½	—	23002	00
9.	6357 at	2	62½	—	16687	12½
10.	1638 at	2	95	—	4832	10
11.	47269 at	4	75	—	224527	75
12.	36296 at	2	62½	—	95277	00
13.	57246 at	5	85	—	334889	10
14.	269423 at	8	96	—	2414030	00
15.	764576 at	4	87½	—	3727308	00
16.	29365 at	3	56¼	—	104612	81

CASE 3.

When the given quantity consists of divers denominations.

RULE.

Multiply the given price by the number of that denomination to which it belongs, and take parts of the price for the inferior denominations.

Note. These cases may be proved by multiplication, or the rule of three.

EXAMPLES.

1. What is the value of 12*C.* 2*qrs.* 16*lb.* of iron at $6 70*cts.* per *cwt?*

q.	*C.*	$	*cts.*
2	½	6	72
			12
16	⅐	80	64
		3	36
			96
		$ 84	96

2. Required the price of 7*T.* 15*C.* of hay at $18 per ton.

c.	*T.*	$	*cts*
10	½	18	00
			7
		126	00
5	½	9	00
		4	50
	Ans.	$ 139	50

or thus

c.	*T.*	$	
5	¼	18	
		8	
		144	
		4	50
	Ans.	$ 139	50

			$	*cts.*
3.	15*C.* 2*qr.* 15*lb.* at $15 16*cts.* per *cwt.*	*Fac.*	236	87½
4.	11*C.* 1*qr.* 16*lb.* at $10 56*cts.* per *cwt.*	—	120	30
5.	9*T.* 15*C.* 2*qrs.* at $24 50*cts.* per ton.	—	239	48
6.	125*a.* 2*r.* 20*p.* at $118 75*cts.* per *ac.*	—	14917	96
7.	15*lb.* 10*oz.* 15*dwts.* at $13 90*cts.* per *lb.*	—	220	95
8.	7*oz.* 10*dwts.* 16*grs.* at $1 16*cts.* per *oz.*	—	8	73
9.	73*yds.* 3*qrs.* 2*na.* at $5 60*cts.* per *yd.*	—	413	70
10.	25*yds.* 1*qr.* at $3 64*cts.* per *yd.*	—	91	91
11.	32*bu.* 3*p.* 6*qts.* at $7 50*cts.* per *bu.*	—	247	04
12.	97*a.* 3*r.* at $26 50*cts.* per acre.	—	2590	37½
13.	59*bu.* 2*p.* 4*qts.* at $9 20*cts.* per *bu.*	—	548	55

14. 5*T.* 12*cwt.* 2*qrs.* 14*lb.* at $2 25cts. per cwt.
Facit $253 40cts.

15. 25*lb.* 10*oz.* 12*dwts.* at $25 50cts. per lb.
Facit $660 02cts.

16. 245*yds.* 3*qrs.* 2*na.* at $9 70cts. per yard.
Ans. $2384 98cts.

17. 5*C.* 23*lb.* of beef at $6 per *cwt.* of *100*lbs.* each. } *Facit* $31 38.

18. 784*a.* 3*r.* 16*p.* at $12 per acre.
Facit $9418 20cts.

19. 256*bush.* 2*p.* 4*qts.* at $2 25cts. per bushel.
Facit $577 40cts.

20. 14*T.* 5*cwt.* 2*qr.* 14*lb.* at $9 60cts per ton.
Facit $137 10cts.

TARE AND TRET.

Tare and tret are allowances sometimes made in the sale of particular commodities.

Gross weight is the weight of the goods, together with the hogshead, barrel, box, &c. in which they are contained.

Tare is an allowance or weight to be deducted from the gross, for the weight of the hogshead, box, bag, &c. containing the commodity.

Tret is an allowance of 4 pounds in 104 for waste, dust, &c.

Neat weight is the true weight of the commodity or what remains after all allowances are deducted.

CASE 1.†

When the tare is a given number of pounds per *cwt.*

RULE.

Subtract from the gross such part or parts thereof as the tare is of an hundred weight.

Or, when the tare cannot be readily found by aliquot parts, it may be computed by the rule of three.

---

* This and other similar questions are commonly best solved by multiplication, the number of cents per pound being equal to the numbers of dollars per *cwt.*

† Writers on arithmetic have generally given two preceding cases. 1. When the whole tare is given. 2. When the tare per barrel, box, &c. is given. But as the former is nothing more than compound subtraction; and the latter an easy exercise in compound multiplication, and both are virtually included in the rule above given, the compiler judged it unnecessary to encumber his work with such minute ramifications.

1. In 75*C*. 3*qrs*. 12*lb*. gross, the tare being 14*lb*. per *cwt*. how much is the neat weight?

	cwt.	qrs.	lb.
⅛)	75	3	12
	9	1	26
	66	1	14 *Ans.*

2. What is the neat weight of 9hhds of sugar, each weighing 6cwt 2qrs 12lb gross; tare 16lb per cwt; and what is the value at $9 per cwt?

Ans. 50cwt 3qrs 25lbs; value $458 75cts.

3. What is the neat weight of 25 barrels of potash, each weighing 1cwt 3qrs 14lb; tare 12lb per cwt?

Ans. 41cwt 3qrs. 11½lb.

4. Required the neat weight of 16hhds. of tobacco, each weighing 15cwt. 3qrs. 16lb. gross; tare 7lb. per cwt., and its value at $28 per cwt.

Ans. weight 238cwt. 1qr. 16lb.
value $6675.

5. What is the neat weight of a quantity of sugar, amounting to 237cwt. 3qrs. 16lb. gross, tare 8lb. per cwt.; and what is its value at $8 50cts per cwt.?

Ans. weight 220cwt. 3qrs. 17lb.
value $1877 66cts.

CASE 2.

When tare and tret are both allowed.

RULE.

Deduct the tare as before, the remainder is called suttle. Divide the suttle by 26, the quotient is the tret, which subtracted from the suttle leaves the neat.

EXAMPLES.

1. In 39 bags of coffee, each weighing 217lb gross; tare 15lb per bag, and tret 4lb in 104; how many pounds neat; and what is the value at 12½cts per pound?

lb.
217
15
——
202 suttle per bag.
39
——
26)7878 whole suttle.
303 tret.
——
|12½|⅛| 7575 neat.
——
$946 87½ value.

2. In 177cwt 0qrs 22lb gross, tare 16lb per cwt; tret as usual, 4lb in 104; how many pounds neat?

Ans. 146cwt 0qrs 5lb.

3. Required the neat weight of 5 casks of currants, each weighing 7cwt 3qrs 17lb gross; tare 2qrs 11lb per cask, and tret 4lb. in 104; also the value at $3 50cts per cwt.

Ans. 35cwt. 13lb. Value $122 92cts.

APPLICATION.

1. What is the neat weight of 38hhds of tobacco, weighing gross 201cwt 3qrs 12lb; tare in the whole 3140lb? *Ans.* 173cwt 3qrs 8lb.

2. Bought 10hhds of sugar, each 10cwt 1qr 14lb gross; tare 7lb per cwt; tret 4lb in 104; what was the cost at $11 20cts per cwt? *Ans.* $1047 50cts.

3. Imported 25hhds of sugar, each weighing 7cwt 3qrs 15lb gross; tare in the whole 6cwt 2qrs 22lb. The price per invoice $1840, freight $37 50cts, duty $41 75cts; what is the cost per cwt of the neat weight?

Ans. $10 08cts.

4. What is the cost of 30 casks of raisins, each weighing 2cwt 3qrs 12lb gross; tare 21lb per cask; at $7 35cts per cwt? *Ans.* $588 65½cts.

5. In 15 bags of coffee, each 5cwt 2qrs 25lb gross, tare per bag 5lb, tret as usual 4lb in 104, how much neat; and what is the value at 14 dollars per cwt?

Ans. Neat 81cwt 3qrs 17lb. Value $1146 62½cts.

6. What will be the price of 27 casks of sugar, each

2cwt 3qrs 14lb gross, tare 6lb per cwt; valued at 15 dollars 60cts per cwt? Ans. $1146 04cts.

7. What is the neat weight and value of 36hhds of rice, each weighing gross 7cwt 2 qrs 16lb, tare 12lb per cwt, and tret 4lb in 104, at 5 dollars 50cts per cwt?

Ans. 236cwt 4qr 25lb. Value $1299 22cts.

INTEREST.

Interest is a consideration paid for the use of money lent or left unpaid after it becomes due.

The *principal* is the sum on which interest is payable.

The *rate per cent. per annum* is the number of dollars* paid for the use of 100 dollars during a year.

The *amount* is the sum of the principal and interest.

When the interest is payable on the first principal only, it is called Simple Interest; but when the interest, as it becomes due, is added to the principal, and interest computed on the amount, it is termed Compound Interest.

SIMPLE INTEREST.

All the cases in Simple Interest are easily solved either by the Single or Double Rule of Three: but there are practical methods which are sometimes more convenient.

When the principal, time, and rate per cent. per annum, are given to find the interest.

GENERAL RULE.†

Multiply the principal by the rate per cent. and divide by 100, the quotient will be the interest for one year. Multiply this interest by the number of years, and take parts for the months, for the interest required.

---

* If accounts are kept in pounds, francs, florins, or any other foreign coin, change the word dollars in this sentence into pounds, francs, florins, &c. and the definition will still be correct.

† When the principal, estimated in dollars, is multiplied by the rate per cent, the product is the interest for one year, in cents.

EXAMPLES.

1. Required the interest of $569 for 3 years and 4 months, at 5 per cent. per annum?

$
569
5

|4|⅓|28 45 interest for one year.
3

85 35
9 48

Ans. $94 83

2. Required the amount of $986 84cts. for 5 years and 6months at 4 per cent per annum?

$ cts.		$ cts.
986 84		986 84 principal.
4		217 08 interest.
|6|½|39 47 36	Ans.	1203 92 amount.
5		
197 35		
19 73		
$217 08 Interest.		

3. What is the interest of $2156 for 2yrs at 6 per cent per annum? Ans. $258 72cts.

4. What is the interest of $598 50cts for 4yrs at 5 per cent per annum? Ans. $119 70cts.

5. What is the amount of a bond for $867 in 2yrs and 9 months at 5 per cent per annum?

Ans. $986 21cts.

6. The British national debt was computed, in the year 1813, to amount to $4030842620 bearing an interest of about 3 per cent per annum; how much is the interest for one hour, reckoning 8766 hours in the year?

Ans. $13794 80cts.

7. Required the amount of 4759 dollars for 3 years and 9 months, at 5 per cent. per annum? Ans. $5651 31cts.

8. What is the interest of 2756 dollars 25cts. for 7 years and 6 months, at 4 per cent. per annum?

Ans. $826 87cts.

9. Quere the amount of 2376 dollars 50cts for 7 years and 4 months, at 4 per cent. per annum?

Ans. $3073 60cts.

When the interest is at 6 per cent per annum, multiply the principal into dollars by half the number of months, the product will be the interest in cents.

* Or, multiply the principal, in dollars, by the number of days, and divide the product by 6 for the interest *nearly* in mills. From this result subtract $\frac{1}{73}$ part thereof, the remainder will be the true interest.

When the interest is computed at 6 per cent. it is readily obtained for any other rate, by taking aliquot parts.

10. What is the interest of $4854 for 18 months, at 6 per cent per annum?

$
4854
9

$436 86 Ans.

11. Required the interest of $3547 for 125 days at 5 per cent per annum.

8	3547,000	*See note page* 36.
6	443,375	
73	73,896 1,01	
6	72,88	Interest at 6 per cent.
	12,14	
Ans.	60,74	Interest at 5 per cent.

---

* This method, without the deduction, is commonly used in computing interest at the banks.—See note on discount, p. 82.

12. What is the amount of 396dolls for 14 months at 6 per cent. per annum? *Ans.* $423 72cts.

13. Calculate the interest due on a bond for 1575dolls, for 20 months, at 4 per cent. per annum. *Facit* $105.

14. What is the interest of 592dolls for 90 days, at 6 per cent. per annum? *Ans.* $8 76cts.

15. What is the amount of a note for 1050dolls, drawn on the 1st of 5th month, and paid on the 17th of the 12th month, computing interest at 6 per cent per annum? *Ans.* $1089 70cts.

16. Required the amount of a bond for 1150 dollars for 18 months, at 6 per cent. per annum. And. $1253 50cts.

17. What is the interest of 4750 dollars for 236 days, at 6 per cent. per annum? Ans. $184 27cts.

18. Quere the amount of 467 dollars 25cts for 93 days, at 6 per cent. per annum? Ans. $474 39cts.

19. Required the amount of 4265 dollars 50cts for 145 days, at 4 per cent. per annum? Ans. $4333 27cts.

REBATE OR DISCOUNT.

Rebate or discount is an allowance made for the payment of money before it becomes due.

The present worth of a debt, payable at a future time, is such a sum as being put to interest, at a given rate per cent. per annum, during the time by which the payment is anticipated, would amount to the given debt; consequently to find the present worth of a debt payable at a given future time, at a proposed rate per cent., is to find the principal, when the amount, time, and rate per cent. per annum, are given.

RULE.*

Find the amount of any principal assumed at pleasure, for the time and rate proposed.

---

* This method of computing discount, is the equitable one; the interest being really estimated at the proposed rate per cent. per annum.—The mode generally adopted at the banks is to compute

Then, As the amount found
Is to the amount given,
So is the principal assumed
To the principal or present worth required.

The present worth subtracted from the amount, will leave the discount. Or,

If the interest of the assumed principal be made the third term in the above analogy, the fourth term will be the discount.

EXAMPLES.

1. What is the present worth of 535dolls, payable in 15 months, allowing discount at 6 per cent. per annum?

	dolls.	
Assumed principal	100	
Int. for 15 months ($\frac{15}{2}$)=	7	50
Amount	107	50

	$	cts.		$		$		$	cts.
As	107	50	:	535	: :	100	:	497	67

2. Required the discount upon a note for 1530dolls paid 18 months before it is due, at 6 per cent per annum? Ans. $126 33cts.

3. Sold goods amounting to 1500dolls, to be paid one half at 9 months, and the other at 18 months; what is the present worth allowing a discount at 5 per cent. per annum? Ans. $1420 56cts.

4. What is the difference between the discount on 1285 dollars for 20 months, at 6 per cent. per annum, and the interest of the same sum for the same time, and at the same rate? $Ans. 11 68cts.

---

the interest on the whole note to be discounted in a manner which produces a small excess, and, deducting this interest, advance the remainder to the holders—Thus virtually charging interest not only on the sum advanced, but on the part withheld. This when the sum is small is a trivial error, but in a large one the error is sometimes considerable. Suppose, for example, a note for $60000 payable in 63 days, to be discounted. The interest as computed at the bank, (see note page 81) is $630, consequently, the holder of the note will receive $59370 in hand;—the true interest of which for 63 days is $614 84cts., differing from the interest charged $15 16cts.

5. What is the present value of a note for 2576 dollars, payable in 9 months, interest being computed at 6 per cent. per annum? Ans. $2465 07cts.

6. Required the discount on a note for 1500 dollars, payable in 63 days, reckoning interest at 6 per cent. per annum? Ans. $15 [illegible]cts. 29

7. Find the present value of 2769 dollars, payable in 18 months, allowing interest at 4 per cent.: and then the amount of that present worth at the same rate, and for the same time. Result. Present value $2612 26cts.

Note.—For the method of computing Compound Interest, see Compound Interest by Decimals.

* * *

INSURANCE, COMMISSION & BROKAGE.

Insurance, Commission and Brokage, are allowances made to insurers, factors, and brokers, at a stipulated rate per cent.

RULE.

Work as for the interest of the given sum, during one year, at the proposed rate per cent.

EXAMPLES.

1. Required the commission on $2436 at 2½ per cent.

```
   $                       $
 2|2436        Or thus 1/40)2436
      2                    ------
   ----                    $60 90cts.
   4872                    ------
   1218
   ----
 $60  90cts.
```

2. What is the insurance of a ship and cargo, valued at $73567 at 12½ per cent? Ans. $9195 87½cts.

3. Required the commission on $3675 at 3 per cent? Ans. $110 25cts.

4. What is the brokage on $25478 at ½ per cent? Ans. $127 39cts.

5. What is the brokage on $3678 at ¾ per cent.?
Ans. $27 58½cts.

EQUATION OF PAYMENTS.

Equation of payments is the method of reducing several stated times at which money is payable, to one mean or equated time, or of finding the times at which several payments shall be made so as to be equivalent to their sum paid at a given time.

RULE.*

Multiply each payment by its time, the sum of the products, divided by the whole debt, will be the equated time.

Or, If the time at which the whole debt becomes payable, be given, and it be proposed to anticipate the payment of a part, to find the time when the remainder must be paid, multiply the whole debt and the part proposed to be anticipated, by their times respectively, and divide the difference of the products by the remaining part of the debt for the time required.

PROOF.

Find the interest of the several payments for their respective times at any assumed rate per cent., their sum will be equal to the interest of the whole debt, at the same rate, for the equated time.

EXAMPLES.

1. F. owes H. $2400, of which $480 are to be paid at 5 months; $960, at 8 months, and the rest at 12 months; but they agree to make one payment of the whole, what is the time?

---

* This rule, though given in substance by most authors on practical arithmetic, is not strictly correct; being founded on the supposition that interest should be computed on the whole debt due at a future time, and not, as it ought to be, on the present worth. Being a near approximation, of easy application, and withal of but little practical importance, it has been admitted in compliance with general usage. The true rule is:—Find the present worth of each instalment, at the legal rate of interest, and find in what time the sum of these present values would amount to the whole debt.

$480\times 5=2400$
$960\times 8=7680$
$960\times 12=11520$

2400)21600(9*months Ans.*
21600

2. A merchant has owing to him $900, to be paid as follows, $150 at 2 months, $300 at 5 months, and $450 at 8 months, what is the equated time for paying the whole debt? *Ans.* 6*mo.*

3. A merchant has purchased goods to the amount of $2000, of which $400 are to be paid at present, $800 at 6 months, and the rest at 9 months ; but it is agreed to make one payment of the whole ; when must it be paid ? *Ans.* 6*mo.*

4. A. owes B. $1200, which are to be paid at the end of 8 months, but A. offers to pay $400 at 4 months on condition that the remainder shall be longer forborne ; what is the time when the rest ought to be paid ? *Ans.* 10*mo.*

5. If a debt of $1000 be payable at the end of 7 months, and the debtor agree to pay $300 at present ; what is the time for paying the rest ? *Ans.* 10*mo.*

◆

THE RULE OF THREE AND PRACTICE.

Exemplified by questions respecting the bartering of goods, and the loss or gain on mercantile transactions.

The word barter is used to denote the exchanging of one commodity for another, and differs from purchase and sale only by the exclusion of cash.

Loss or gain is commonly estimated at so much per cent.

The gain or loss per cent is the number of dollars, pounds, francs, &c. gained or lost on every 100 dollars, pounds, francs, &c. contained in the *prime cost.**

---

* The learner is requested to attend carefully to this definition. Some arithmetical writers have fallen into considerable errors by computing the gain or loss per cent on the selling price instead of the prime cost.

EXAMPLES.

1. How many dozen of candles at 18*cts.* per dozen must be given in barter for 12*C.* 2*qrs.* 14*lb.* of tallow at $14 12 *cts.* per *cwt* ?

$$\begin{array}{lrr}
\textit{qrs.} & \$ & \textit{cts.} \\
2|\tfrac{1}{2}| & 14 & 12 \\
 & & 12 \\ \cline{2-3}
\textit{lb.} & 169 & 44 \\
14|\tfrac{1}{4}| & 7 & 06 \\
 & 1 & 76 \\ \cline{2-3}
 & 178 & 26 = \text{price of the tallow.}
\end{array}$$

$$\begin{array}{lclclcl}
\textit{cts.} & & \$\ \textit{cts.} & & \textit{lb.} & & \textit{lb.} \\
\text{As } 18 & : & 178\ 26 & :: & 1 & : & 990\ \textit{Ans.}
\end{array}$$

2. How many pounds of tea at $2 50*cts.* per pound must be given in barter for 9*C.* 3*qrs.* 21*lb.* of sugar at $9 26*cts.* per *cwt* ? *Ans.* 36*lb.*

3. How many bushels of wheat at $1 25*cts.* per bushel must be given for 150*bu.* of rye at 70*cts.* per bushel ? *Ans.* 84 *bushels.*

4. L. has 41*cwt.* of hops at $9 50*cts.* per *cwt.* which he barters with M. for salt at 75*cts.* per bushel, how much salt must he receive ? *Ans.* 519*bu.* 1*p.*+

5. C. has wheat at $1 25*cts.* per bushel, if paid for in cash, but in barter demands $1 50*cts.* per bushel ; D. has cotton at 20*cts.* per pound ready money ; what price should the cotton be in barter, and how much cotton must be given for 124 bushels of wheat ? *Ans.* 24*cts.* per *lb. and* 775*lb.*

6. P. has coffee which he barters with Q. at 5*cts.* per pound more than it cost him, against tea which cost $1 25 *cts.* per pound, but is raised to $1 50*cts.* per pound. Quere the prime cost of the coffee ? *Ans.* 25*cts.*

7. If 18*Cwt.* of iron be purchased at $6 50*cts.* per *Cwt.* and sold at 7*cts.* per pound ; how much is gained by the transaction ? *Ans.* $24 12*cts.*

8. If hats be purchased at $4, and sold at $4 50*cts.* what is the gain per cent ? *Ans.* 12½.

9. If 125 yards of cloth cost $397 50*cts*, how must it be sold per yard to gain $40 by the whole ? *Ans.* $3 50*cts.*

10. Bought 60 reams of paper, at $4 per ream; what is lost in the whole quantity at 4 per cent? *Ans.* $9 60*cts.*

11. Sold 200 penknives at 18cts each, by which I lost 10 per cent; how much is lost in the whole? Ans. $4.

12. If 300lb of coffee be purchased at 25cts per pound, ready money, and sold for 30cts per pound, payable in 8 months; how much will be gained by the transaction, allowing discount at 6 per cent per annum, and how much per cent per annum?

Ans. $11 53cts. gain 30 per ct. per annum.

13. If by selling muslin at 45cts per yard I gain 12½ per cent. what was the prime cost of a yard?

Ans. 40cts.

14. If a chest of tea, weighing 490lb. be bought for $326, and sold for $370 10cts.; what was the profit on each pound? Ans. 9cts.

15. Sold cloth at $11 50cts per yard, and thereby gained 15 per cent.; what would have been gained per cent if it had been sold at $12 per yard? Ans. 20.

16. If by selling pepper at 10½cts. per pound there are 2cts. lost on each; required the loss per cent?

Ans. 16.

17. A person about to purchase 1000 gallons of molasses, finds the price to be 50cts per gallon, ready money, or 52cts. payable in two months; how much will he gain or lose by borrowing money at 6 per cent. per annum, to avail himself of the cash price? Ans. $15 gain.

18. A trader, being embarrassed, owes 3400 dollars, which the creditor requires to be immediately paid; he has goods which he can sell at auction, for cash at 15 per cent below the just value; he cannot borrow money without allowing a premium of 9 per cent. and paying interest at 6 per cent. per annum on the whole. Now, admitting he can sell his goods for their value within a year, which will be more eligible, to send them to auction, or to borrow money on these conditions, to satisfy his creditor?

Ans. To borrow, by $71 64cts.

VULGAR FRACTIONS.

A fraction is a part or parts of a unit. Fractions are either vulgar or decimal.

In vulgar fractions the unit may be considered as divided into any number whatever of equal parts.

The denominator indicates the number of parts into which the unit is supposed to be divided.

The numerator expresses the number of those parts denoted by the fraction; thus in the fraction $\frac{2}{3}$ (read two thirds) the denominator is 3, and indicates that the unit is divided into 3 equal parts, and the numerator 2 shows that the fraction expresses two of those parts.*

A proper fraction is that in which the numerator is less than the denominator, as $\frac{5}{6}$.

An improper fraction is one in which the numerator equals or exceeds the denominator, as $\frac{3}{3}$ or $\frac{9}{7}$

A mixed number consists of a whole number and a fraction, as $5\frac{3}{8}$.

A compound fraction is a fraction of a fraction, as $\frac{2}{3}$ of $\frac{7}{8}$.

A mixed fraction is one whose numerator or denominator is a mixed number as $\dfrac{42\frac{3}{4}}{49}$ or, $\dfrac{15}{23\frac{1}{2}}$

The reciprocal of a fraction is the fraction inverted, thus $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$, and $\frac{1}{3}$ the reciprocal of 3, or of $\frac{3}{1}$.

The two following problems are not fractional, but are useful in the solution of some cases of Reduction of Vulgar Fractions.

PROBLEM 1.

To find the greatest common measure of two given numbers. (See definition 10, page 14.)

RULE.

Divide the greater number by the less, and the last divisor by the remainder, till nothing remain, the last divisor will be the common measure or divisor sought.

Note.—To find the greatest common measure of three or more given numbers; find the common measure of two of them, as above, then find the greatest divisor of this common measure and a third given number; and so proceed to the last.

---

* Hence to express a whole number fractionally, put a unit for for its denominator, as $5=\frac{1}{5}$.

EXAMPLES.

1. Required the greatest common measure of 135, 165, and 235.

```
135)165(1
    135
    ———
     30)135(4
        120
        ———
         15)30(2
            30
            ——
```

Hence the common measure of 135 and 165 is 15.

Again the common measure of 15 and 235 found as before, is 5. The answer sought.

2. What is the greatest common measure of 323 and 425? *Ans*. 17.

3. Required the greatest common measure of 2310 and 4626? *Ans*. 6.

4. What is the greatest common divisor of 1092, 1428, 1197 and 805? *Ans*. 7.

PROBLEM 2.

To find the least common multiple of two or more given numbers. (See definitions 11 and 12, page 14.)

RULE.

Divide two or more of the given numbers by any *prime** number that will divide them, repeat the operation on their quotients and the undivided numbers, until they become prime to each other; multiply the resulting numbers and the several divisors continually together for the common multiple required.

Note. If the given numbers are prime to each other, their continued product is their least common multiple.

EXAMPLES.

1. Required the least common multiple of 12, 25, 30, and 45?

---

* The experienced arithmetician need not always confine himself to prime divisors. When composite divisors are used, an unskilful computer is liable to obtain a multiple which is not the least possible.

3	12	25	30	45
5	4	25	10	15
2	4	5	2	3
	2	5	1	3

3 × 5 × 2 × 2 × 5 × 3 = 900, the common multiplier required.

Note. If, in this example, I had divided by 6 instead of the prime numbers 3 and 2, the result would have been 2700, because 45 which is a multiple of 3 is not divisible by 6.

2. Required the least common multiple of 12, 16, 20 and 30. *Ans.* 240.
3. What is the least common multiple of 25, 35, 60 and 72? *Ans.* 12600.
4. What is the least common multiple of 105, 140, 175, and 245? *Ans.* 14700.
5. Required the least common multiple of 108, 60, 63, and 132. Result 41580.

REDUCTION OF VULGAR FRACTIONS.

CASE 1.

To reduce a vulgar fraction to its lowest terms.

RULE.

Divide the numerator and denominator by their greatest common measure.

EXAMPLES.

1. Reduce $\frac{125}{650}$ to its lowest terms.

The greatest common measure, found by problem 1st. is 25

$25)\frac{125}{650} = \frac{5}{26}$. the fraction sought.

2. Reduce $\frac{91}{119}$ to its lowest terms. *Result* $\frac{13}{17}$.
3. Reduce $\frac{195}{468}$ to its lowest terms. *Result* $\frac{5}{12}$.

4. Required the value of $\frac{1776}{1887}$ in its lowest terms?

Ans. $\frac{16}{17}$.

5. Required the value of $\frac{417}{973}$ in its lowest terms?

Ans. $\frac{3}{7}$.

6. Reduce $\frac{896}{1152}$ to its lowest terms. *Result* $\frac{7}{9}$.

CASE 2.

To reduce any given fractions to other equivalent ones, having a common denominator.

RULE.

Reduce the given fractions to their lowest terms, then multiply each numerator by all the denominators but its own for a new numerator; and all the denominators together for a common denominator. Or.

Find the least common multiple of the given denominators, by problem 2, then divide this multiple by each of the given denominators, and multiply the quotients severally by the correspondent numerators for the numerators required, under which write the common multiple, for the least possible common denominator.

Note 1.—When the given denominators are prime to each other, these two methods produce the same result; in other cases the latter is preferable to the former.

2. This case is proved by the former.

EXAMPLES.

1. Reduce $\frac{6}{8}$, $\frac{15}{36}$, $\frac{7}{9}$, and $\frac{5}{16}$ to equivalent fractions having a common denominator.

$\frac{6}{8}=\frac{3}{4}$, $\frac{15}{36}=\frac{5}{12}$. $4\times12\times9\times16=6912.$=com. denom.

$$\begin{array}{cccc}
3 & 5 & 7 & 5 \\
12 & 4 & 4 & 4 \\
\hline
36 & 20 & 28 & 20 \\
9 & 9 & 12 & 12 \\
\hline
324 & 180 & 336 & 240 \\
16 & 16 & 16 & 9 \\
\hline
\frac{5184}{6912} & \frac{2880}{6912} & \frac{5376}{6912} & \frac{2160}{6912} \\
\hline
\end{array}$$

the fractions required.

Or, The least common multiple of 8, 36, 9 and 16 is 144.

8)144	36)144	9)144	16)144
18	4	16	9
6	15	7	5
$\frac{108}{144}$	$\frac{60}{144}$	$\frac{112}{144}$	$\frac{45}{144}$

The fractions sought with the least possible common denominator.

2. Reduce $\frac{5}{9}$, $\frac{3}{7}$, and $\frac{5}{16}$ to equivalent fractions having a common denominator. Result $\frac{560}{1008}$, $\frac{432}{1008}$, $\frac{315}{1008}$.

3. Reduce $\frac{5}{7}$, $\frac{9}{14}$, $\frac{5}{21}$, and $\frac{8}{42}$ to equivalent fractions, having the least possible common denominator. Result $\frac{30}{42}$, $\frac{27}{42}$, $\frac{10}{42}$, $\frac{8}{42}$.

4. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$, to a common denominator. Result $\frac{72}{144}$, $\frac{96}{144}$, $\frac{108}{144}$, and $\frac{120}{144}$, Or, $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$

5. Reduce $\frac{7}{11}$, $\frac{5}{9}$, $\frac{8}{14}$, and $\frac{15}{22}$, to the equivalent fractions with the least common denominator possible. Result. $\frac{882}{1386}$, $\frac{770}{1386}$, $\frac{792}{1386}$, $\frac{945}{1386}$.

CASE 3.

To reduce a mixed number to an improper fraction.

RULE.

Multiply the whole number by the denominator of the fraction, and add the numerator to the product for the numerator of the fraction sought, under which write the given denominator.

EXAMPLES.

1. Reduce $5\frac{6}{7}$ to an improper fraction.

$$\begin{array}{r} 5\frac{6}{7} \\ 7 \\ \hline \frac{41}{7} \end{array}$$

2. Reduce $15\frac{7}{11}$ to an improper fraction. Result $\frac{172}{11}$.
3. Reduce $17\frac{5}{21}$ to an improper fraction. Result $\frac{362}{21}$.
4. Reduce $23\frac{17}{35}$ to an improper fraction. Result $\frac{822}{35}$.
5. Reduce $71\frac{13}{62}$ to an improper fraction. Result $\frac{4415}{62}$.

CASE 4.

To reduce an improper fraction to a whole or mixed number.

RULE.

Divide the numerator by the denominator, set down the quotient for the whole number, and the remainder, if any, with the given denominator placed under it for the fractional part.

Note. This case and the one immediately preceding prove each other.

EXAMPLES.

1. Reduce $\frac{4415}{62}$ to a whole or mixed number.

$$\begin{array}{r} 62)4415(71\frac{13}{62} \text{ Result.} \\ 434\phantom{(71\frac{13}{62} \text{ Result.}} \\ \hline 75\phantom{(71\frac{13}{62} \text{ Result.}} \\ 62\phantom{(71\frac{13}{62} \text{ Result.}} \\ \hline 13\phantom{(71\frac{13}{62} \text{ Result.}} \\ \hline \end{array}$$

2. Reduce $\frac{354}{27}$ to an equivalent whole or mixed number. *Result* $13\frac{1}{9}$.

3. Reduce $\frac{577}{39}$ to a whole or mixed number. *Result* $14\frac{31}{39}$.

4. Reduce $\frac{3446}{47}$ to a whole or mixed number. *Result* $73\frac{15}{47}$.

5. Required the equivalent whole or mixed number for $\frac{2505}{19}$. *Ans.* $131\frac{16}{19}$,

CASE 5.

To reduce a compound fraction to a single one.*

RULE.

Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

Note. When the numerator of one fraction is the same as the denominator of another, they may be cancelled; and others frequently abridged by taking aliquot parts.

---

* The three following cases are only particular applications of this case.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ to a single fraction.

$\frac{2}{3}\times\frac{3}{4}\times\frac{5}{6}=\frac{30}{72}=\frac{5}{12}$. Or,

$$\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\underset{2}{\cancel{4}}}\times\frac{5}{6}=\frac{5}{12}\ \textit{Result.}$$

2. Required the value of $\frac{3}{4}$ of $\frac{9}{10}$ of $\frac{15}{16}$ in a single fraction? *Ans.* $\frac{81}{128}$.

3. Reduce $\frac{5}{9}$ of $\frac{7}{10}$ of $\frac{15}{19}$ to a single fraction. *Result* $\frac{35}{114}$.

4. Reduce $\frac{10}{11}$ of $\frac{11}{12}$ of $\frac{24}{29}$ to a single fraction. *Result* $\frac{20}{29}$.

5. Reduce $\frac{15}{17}$ of $\frac{17}{30}$ of $\frac{4}{5}$ to a single fraction. *Result* $\frac{2}{5}$.

CASE 6.

To reduce the fraction of one denomination to the fraction of another, either greater or less, retaining the original value.

RULE.

Make it a compound fraction by comparing it with all the denominations between it and that to which it is to be reduced, and reduce this compound fraction to a single one.

EXAMPLES.

1. Reduce $\frac{8}{9}$ of a *dwt.* to the fraction of a *lb.*

$$\frac{\cancel{8}\cancel{2}}{9}\text{ of }\frac{1}{\underset{10}{\cancel{20}}}\text{ of }\overset{lb.}{\frac{1}{\underset{3}{\cancel{12}}}}=\overset{lb.}{\frac{1}{270}}\ \textit{Result.}$$

2. Reduce $\frac{1}{1584}$ of a day to the fraction of a minute.

$$\frac{1}{1584}\text{ of }\overset{min.}{\frac{24}{1}}\text{ of }\overset{min.}{\frac{60}{1}}=\frac{1440}{1584}=\overset{m.}{\frac{10}{11}}\ \textit{Result.}$$

3. Reduce $\frac{5}{6}$ of a penny to the fraction of a pound. *Result.* $\frac{1}{288}$£.

4. Reduce $\frac{9}{13}$ of a pint of wine to the fraction of a *hhd.* *Result* $\frac{1}{728}$*hhd.*

5. Reduce $\frac{2}{27}$ of a *lb.* troy to the fraction of an *oz.* *Result* $\frac{8}{9}$*oz.*

6. Reduce $\frac{3}{392}$ of an *cwt.* to the fraction of a *lb.*
Result $\frac{6}{7}$*lb.*

7. Reduce $\frac{1}{2}$ of a second to the fraction of an hour.
Result $\frac{1}{7200}$hrs.

8. Reduce $\frac{7}{85}$ of a week to the fraction of a day.
Result $\frac{49}{85}$d.

9. Reduce $\frac{5}{7}$ of a yard to the fraction of a mile.
Result $\frac{1}{2464}$m.

10. Reduce $\frac{7}{5760}$ of a furlong to the fraction of a foot.
Result $\frac{77}{96}$ft.

11. What part of a chaldron of coals is $\frac{3}{4}$ of a bushel?
Ans. $\frac{1}{48}$ch.

12. What fraction of a lb. is equivalent to $\frac{3}{9865}$ of a ton?
Ans. $\frac{1344}{1973}$lb.

CASE 7.

To reduce a fraction to the known parts of an integer.

RULE.

Reduce the given fraction to one of the next inferior denomination, by the last case; if the result be an improper fraction reduce it to a mixed number, by case 4; reduce the fractional part as before, and so proceed to the lowest denomination.

EXAMPLES.

1. What is the proper value of $\frac{2}{3}$ of a £ sterling?

$\frac{2}{3}$ of $\frac{20}{1}$s. $=\frac{40}{3}$s. $=13\frac{1}{3}$s. *Ans.* 13s. 4d.

$$\frac{1}{\cancel{3}} \text{ of } \frac{\overset{\text{d.}}{\cancel{12}^{4}}}{1} = \overset{\text{d.}}{4}.$$

2. Required the value of $\frac{3}{20}$ of a dollar? *Ans.* 15cts.

3. What is the distance of $\frac{9}{10}$ of a mile? *Ans.* 7fur. 8p.

4. Required the value of $\frac{3}{7}$ of a ton?
Ans. 8C. 2qrs. 8lb.

5. Required the value of $\frac{3}{10}$ of a day?
Ans. 7hrs. 12min.

6. Required the value of $\frac{5}{19}$ of an acre?
Ans. 1r. 2p. $\frac{2}{19}$

7. What is the sterling value of $\frac{2}{9}$ of an English guinea?
Ans. 4s. 8d.

8. Reduce $\frac{19}{20}$ of an ell English to its proper value.

Result. 1yd. 0qrs. 3na.

9. What is the value of $\frac{3}{5}$ of $\frac{5}{8}$ of a dollar?

Ans. $37\frac{1}{2}$ cts.

10. Required the value of $\frac{9}{13}$ of 10C. 2qrs.

Ans. 7C. 1qr. $2\frac{2}{13}$lb.

CASE 8.

To reduce a given quantity to the fraction of an integer of an higher denomination.

RULE.

Reduce the given quantity to its lowest denomination mentioned, for a numerator, and the integer to the same denomination for a denominator; and reduce this fraction to its lowest terms. Or,

Reduce the given quantity as above, and compound it with the numbers in the several denominations, between that to which it is reduced, and the integer proposed; and reduce the compound fraction to a single one.

Note.—If there is a fraction annexed to the given number, reduce the mixed number to an improper fraction, and proceed as above.

2. This case and the last prove each other.

EXAMPLES.

1. Reduce 3qrs. 2na to the fraction of a yard.

$$\begin{array}{ccc} \text{qrs.} & \text{na.} & \text{qrs.} \\ 3 & 2 & 4 \\ 4 & & 4 \\ \hline 14\text{na.} & & 16\text{na.}\ \frac{14}{16}=\frac{7}{8}\text{yd.} \\ \hline \end{array}$$

$$\text{Or, } \frac{14}{1} \text{ of } \frac{1}{4} \text{ of } \frac{1}{4} = \frac{14}{16} = \overset{\text{yd.}}{\frac{7}{8}}$$

2. Reduce $5\frac{1}{43}$ pence to the fraction of a shilling.

$$\begin{array}{l} 5\frac{1}{43} \\ 43 \\ \hline \frac{216}{43} \text{ of } \overset{\text{s.}}{\frac{1}{12}} = \overset{\text{s.}}{\frac{216}{516}} = \overset{\text{s.}}{\frac{18}{43}}. \end{array}$$

Result.

3. Reduce $12\frac{1}{2}$ cts. to the fraction of a dollar.
Result $\frac{1}{8}$\$

4. Reduce $56\frac{1}{4}$ cts. to the fraction of a dollar.
Result $\frac{9}{16}$\$

5. Reduce 3C. 2qrs. 14lb. to the fraction of a ton.
Result $\frac{29}{160}$ ton.

6. Reduce 13hrs. 30min. to the fraction of a day.
Result $\frac{9}{16}$d.

7. Reduce 3C. 8lb. 9oz. $13\frac{7}{13}$drs. to the fraction of a ton.
Result $\frac{2}{13}$ton.

CASE 9.

To reduce a given fraction to another equivalent one, having a given numerator.

RULE.

As the numerator of the given fraction
Is to the denominator,
So is the numerator of the proposed fraction
To its denominator.

Or, Multiply the proposed numerator by the denominator of the given fraction and divide by the numerator, the result will be the denominator sought.

EXAMPLES.

1. Reduce $\frac{3}{4}$ to a fraction of the same value, the numerator of which shall be 15.

As 3 : 4 : : 15 : 20. *denom.* or $\frac{15}{1} \times \frac{4}{3} = 20$. *Result* $\frac{15}{20}$.

2. Reduce $\frac{7}{8}$ to a fraction of the same value, having its numerator 42.
Result $\frac{42}{48}$

3. Reduce $\frac{8}{9}$ to an equivalent fraction, the numerator of which shall be 27.

Result $\dfrac{27}{30\frac{3}{8}}$

4. If $\frac{5}{16}$ be reduced to an equivalent fraction, having its numerator 36; what will that fraction be?

Ans. $\dfrac{36}{115\frac{1}{5}}$

CASE 10.

To reduce a given fraction to another equivalent one, having a given denominator.

RULE.

As the denominator of the given fraction,
Is to its numerator,
So is the denominator of the intended fraction
To the numerator required.

Or, Multiply the proposed denominator by the numerator of the given fraction and divide by the denominator, the result will be the numerator sought.

EXAMPLES.

1. Reduce $\frac{7}{8}$ to an equivalent fraction, having its denominator 24.

As 8 : 7 : : 24 : 21 *num.* or $\frac{24}{1} \times \frac{7}{8} = 21$ *Result* $\frac{21}{24}$.

2. Reduce $\frac{5}{9}$ to a fraction of the same value, the denominator of which shall be 45. *Result* $\frac{25}{45}$

3. Reduce $\frac{4}{17}$ to an equivalent fraction having its denominator 68. *Reuslt* $\frac{16}{68}$

4. Reduce $\frac{3}{4}$ to a fraction of the same value, the denominator of which shall be 46. *Result* $\dfrac{34\frac{1}{2}}{46}$

5. Reduce $\frac{7}{11}$ to an equivalent fraction having its denominator 20. *Result* $\dfrac{12\frac{8}{11}}{20}$

CASE 11.

To reduce a mixed fraction to a simple one.

RULE.

Multiply the numerator and denominator of the given fraction by the denominator of the fraction annexed, the products will be the terms of the fraction required.*

EXAMPLES.

1. Reduce $\dfrac{42\frac{7}{8}}{49}$ to a simple fraction.

$$\frac{42\frac{7}{8}}{49} \times \frac{8}{8} = \frac{343}{392} = \frac{7}{8} \text{ Result.}$$

---

* In the application of this rule, it should be considered that a fraction multiplied by a whole number equal to its denominator produces a whole number equal to its numerator.

2. Reduce $\frac{34\frac{1}{2}}{49}$ to a simple fraction. *Result* $\frac{3}{4}$.

3. Reduce $\frac{34}{45\frac{1}{3}}$ to a simple fraction. *Result* $\frac{3}{4}$.

4. Reduce $\frac{73}{131\frac{2}{5}}$ to a simple fraction. *Result* $\frac{5}{9}$.

CASE 12.

To find approximate values of a given fraction, in smaller terms than the fraction accurately expressed will admit.

RULE.

Divide the greater term by the less, and the divisor by the remainder, &c. as in problem 1st.

Then, if the given fraction be a proper one, assume a unit for the numerator, and the first quotient, found as above, for the denominator of the first approximate value.

Multiply the terms of this fraction, by the second quotient, adding 1 to the product of the denominator, for the corresponding terms of the second approximate value.

For any subsequent approximation : multiply the terms of the fraction last found, by the following quotient, and add the corresponding terms of the preceding fraction for the terms of the fraction required.

If the given fraction be an improper one, the reciprocals of the several fractions, found as above directed, will be the fractions required.

EXAMPLES.

1. Required the approximate values of $\frac{100000}{314159}$?

100000)314159(3
300000

14159)100000(7
99113

887)14159(15
13305

854)887(1
854

—— &c.

$$\frac{1}{3} \text{ first approximate value.}$$

$$\frac{1 \times 7}{3 \times 7 + 1} = \frac{7}{22} \text{ 2d app. value.}$$

$$\frac{7 \times 15 + 1}{22 \times 15 + 3} = \frac{106}{333} \text{ 3d do.}$$

$$\frac{106 \times 1 + 7}{333 \times 1 + 22} = \frac{113}{355} \text{ 4th do.}$$

2. Required the approximate values of $\frac{392699}{500000}$.

Ans. $\frac{1}{1}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{9}$, $\frac{11}{14}$, $\frac{172}{219}$ &c.

3. What are the approximate values of $\frac{583671}{2387426}$?

Ans. $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{5}{12}$, $\frac{8}{19}$, $\frac{53}{126}$ &c.

4. Required the approximate values of $\frac{87968}{365256}$.

Ans. $\frac{1}{4}$, $\frac{6}{25}$, $\frac{7}{29}$, $\frac{13}{54}$, $\frac{33}{137}$, &c.

5. What are the approximate values of $\frac{224700}{365256}$?

Ans. $\frac{8}{13}$, $\frac{235}{382}$, $\frac{713}{1159}$, &c.

Note.—The numbers in the 1st and 2nd examples express nearly the proportion of a circle's diameter to its circumference, and of its area to the circumscribing square respectively.

◆

ADDITION OF VULGAR FRACTIONS.

RULE.

Simple fractions, belonging to the same integer, and having a common denominator, are added, by taking the sum of the numerators, and placing the common denominator under it.

If the fractions have different denominators, reduce them to a common denominator, then add as before.

Compound fractions must be reduced to single ones.

Fractions of different integers must be reduced to their proper values, and added as in compound addition; or to fractions of a common integer, and added as above.

EXAMPLES.

1. What is the sum of $\frac{3}{8}$, $\frac{7}{8}$, $\frac{5}{8}$, and $\frac{1}{8}$?

$\frac{3}{8}+\frac{7}{8}+\frac{5}{8}+\frac{1}{8}=\frac{16}{8}=2$ Ans.

2. Add $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{3}{4}$ of $\frac{7}{8}$, and $\frac{5}{16}$ together.

$\frac{1}{2}$ of $\frac{2}{3}=\frac{1}{3}$, $\frac{3}{4}$ of $\frac{7}{8}=\frac{21}{32}$, $\frac{1}{3}$, $\frac{21}{32}$, $\frac{5}{16}$

$\frac{32}{96}+\frac{63}{96}+\frac{30}{96}=\frac{125}{96}=1\frac{29}{96}$ Sum.

3. Add $\frac{7}{15}$ of a pound troy to $\frac{7}{12}$ of an ounce.

oz.
$\frac{7}{15}$ of $\frac{12}{1}$ $=$5oz. 12dwts.
$\frac{7}{12}$ of $\frac{20}{1}$ dwts$=$ 11dwts. 16grs.

6oz. 3dwts. 16grs. Sum.

oz. oz. oz. oz. oz. oz.
Or thus, $\frac{7}{15}$ of $\frac{12}{1}=\frac{28}{5}$, $\frac{28}{5}+\frac{7}{12}=\frac{336}{60}+\frac{35}{60}=\frac{371}{60}=$6oz. 3dwts. 16grs.

4. Required the sum of $\frac{5}{9}$, $\frac{7}{9}$, $\frac{8}{9}$, and $\frac{2}{9}$? Ans. $2\frac{4}{9}$.
5. What is the sum of $\frac{2}{15}$, $\frac{7}{15}$, $\frac{4}{15}$, and $\frac{11}{15}$? Ans. $1\frac{9}{15}$.
6. What is the sum of $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$, and $\frac{11}{12}$? Ans. $3\frac{7}{18}$.
7. Required the sum of $\frac{5}{7}$, $\frac{3}{8}$, $\frac{9}{10}$, and $\frac{8}{15}$. Ans. $2\frac{439}{840}$.
8. Add $\frac{9}{11}$, $\frac{5}{7}$, $4\frac{3}{4}$, and $\frac{3}{4}$ of $\frac{5}{9}$ together. Result $6\frac{323}{462}$.
9. Add $\frac{1}{3}$ of 95, and $\frac{7}{8}$ of 14 together. Result $43\frac{11}{12}$.
10. Add $12\frac{1}{2}$, $3\frac{2}{3}$, and $4\frac{3}{4}$ together. Result $20\frac{11}{12}$.
11. Add $\frac{3}{11}$ of a dollar, $\frac{7}{15}$ of a dime, and $\frac{4}{7}$ of an eagle together. Result \$6 $3\frac{85}{231}$cts.
12. Add $\frac{7}{9}$ of a ton, to $\frac{9}{10}$ of a cwt.
Result 16cwt. 1qr. $23\frac{1}{45}$lb.
13. Add $\frac{2}{5}$ of a day to $\frac{5}{6}$ of an hour.
Result 10hrs. 26min.
14. Add $\frac{5}{9}$ of a week, $\frac{7}{10}$ of a day, and $\frac{6}{7}$ of an hour together. Result 4 days 14hrs. $59\frac{3}{7}$min.
15. What is the sum of $\frac{2}{7}$ of £15, £$3\frac{3}{7}$, $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of a £., and $\frac{2}{3}$ of $\frac{3}{7}$ of a shilling? Ans. £7 17*s.* $5\frac{1}{7}$*d.*
16. If a merchant own $\frac{3}{8}$ of a ship, valued at \$4000, and buy another person's share, which is $\frac{5}{16}$; what part belongs to him, and what is it worth?
Ans. $\frac{11}{16}$; worth \$2750.
17. What is the sum of $\frac{3}{4}$ of $\frac{5}{6}$ of \$15, $\frac{7}{8}$ of $\frac{5}{14}$ of \$25, and $\frac{9}{10}$ of $\frac{10}{23}$ of $\frac{5}{11}$ of $\$13\frac{4}{5}$? Ans. $\$19\frac{113}{176}$

18. Required the sum of $\frac{3}{4}$ of $\frac{5}{6}$ of $3\frac{1}{2}$ tons, $\frac{2}{3}$ of $\frac{9}{10}$ of $2\frac{1}{2}$ tons, and $\frac{5}{6}$ of $\frac{7}{10}$ of $5\frac{3}{4}$cwt ?

Result 3T. 17cwt. $11\frac{2}{3}$lb.

19. Add $\frac{3}{8}$ of $\frac{5}{6}$ of $1\frac{3}{5}$ acres, $\frac{3}{7}$ of $\frac{1}{2}$ of $2\frac{1}{2}$ acres, $\frac{5}{9}$ of $\frac{3}{5}$ of $7\frac{1}{2}$ acres, and $\frac{2}{3}$ of $\frac{11}{12}$ of $3\frac{3}{5}$ acres together.

Result $5\frac{7}{10}$ acres.

SUBTRACTION OF VULGAR FRACTIONS.

RULE.

Prepare the given fractions, when necessary, as in addition, and take the difference of the numerators, under which write the common denominator.

Note.—When mixed numbers are given to be subtracted, it will frequently be convenient to reduce them to improper fractions; and then proceed as directed in the rule.

EXAMPLES.

1. From $\frac{9}{13}$ take $\frac{4}{13}$. $\frac{9}{13}-\frac{4}{13}=\frac{5}{13}$ Result.

2. What is the difference between $17\frac{4}{15}$ and $8\frac{25}{27}$?

$17\frac{4}{15}=\frac{259}{15}$; $8\frac{25}{27}=\frac{241}{27}$; $\frac{259}{15}-\frac{241}{27}=\frac{2331}{135}-\frac{1205}{135}=\frac{1126}{135}=8\frac{46}{135}$.

3. From $\frac{17}{19}$ take $\frac{5}{19}$. Result $\frac{12}{19}$.

4. From $\frac{18}{27}$ take $\frac{1}{3}$. Result $\frac{1}{3}$.

5. Required the difference between $\frac{35}{49}$ and $\frac{15}{35}$.

Ans. $\frac{2}{7}$.

6. What is the excess of $\frac{15}{19}$ above $\frac{3}{15}$? Ans. $\frac{168}{285}$.

7. From $14\frac{3}{7}$ take $12\frac{3}{5}$. Ans. $1\frac{29}{35}$.

8. From $\frac{2}{3}$ of $\frac{5}{9}$ take $\frac{3}{7}$ of $\frac{2}{11}$. Result $\frac{608}{2079}$.

9. From $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{11}{12}$ take $\frac{2}{5}$ of $\frac{5}{7}$ of $\frac{9}{11}$. Result $\frac{31}{385}$.

10. What is the difference between $\frac{9}{10}$ of a ton, and $\frac{11}{12}$ of $\frac{13}{15}$ of $2\frac{3}{4}$cwt.? Ans. 15cwt. 3qrs. $7\frac{14}{45}$lb.

11. From $103\frac{7}{8}$ acres take $\frac{3}{7}$ of $\frac{11}{19}$ of $70\frac{1}{6}$ acres.

Result 71ac. 1r. $30\frac{259}{399}$per.

12. What is the difference between $\frac{2}{5}$ of $\frac{5}{6}$ of \$15, and $\frac{3}{4}$ of $\frac{4}{9}$ of \$$2\frac{1}{2}$? *Ans.* \$$4\frac{1}{4}$.

13. From $13\frac{2}{3}$C. take $\frac{2}{3}$ of $\frac{5}{9}$ of $3\frac{3}{8}$C. *Result* $12\frac{5}{12}$.C.

14. What is the difference between $\frac{5}{9}$ of $\frac{8}{15}$ of $3\frac{1}{4}$lbs. Troy, and $\frac{7}{16}$ of $\frac{4}{9}$ of $3\frac{3}{8}$lbs.? *Ans.* 3oz. 13dwts. $14\frac{2}{3}$ gr.

15. From the sum of $\frac{5}{9}$ of 7 miles, $\frac{3}{8}$ of $\frac{7}{8}$ of $2\frac{1}{4}$ m. and $\frac{5}{6}$ of $\frac{3}{10}$ of $3\frac{1}{2}$m. take the sum of $\frac{5}{7}$ of $\frac{3}{5}$ of $1\frac{3}{4}$m., $\frac{8}{9}$ of $\frac{9}{10}$ of $2\frac{1}{5}$ m., and $\frac{7}{11}$ of $\frac{5}{7}$ of $2\frac{2}{3}$m. *Result* 2m. 2f. $12\frac{1}{4}\frac{3}{5}$p.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.

Reduce mixed numbers to improper fractions and mixed fractions to single ones, then multiply the numerators and denominators of the given factors respectively together for the numerator and denominator of the product required.

Note.—The operations may be sometimes abridged, by cancelling like numbers in the numerator and denominator, or by substituting their aliquot parts.

EXAMPLES.

1. What is the product of $3\frac{4}{5}$ and $\frac{3}{5}$ of $\frac{5}{7}$ of $\frac{14}{19}$?

$$\begin{array}{l} 3\frac{4}{5} \\ 5 \\ \hline \end{array}$$

$\frac{19}{5} \times \frac{3}{5}$ of $\frac{5}{7}$ of $\frac{14}{19} = \frac{3990}{3325} = \frac{6}{5} = 1\frac{1}{5}$ *Ans.*

Or thus $\frac{\cancel{19}}{\cancel{5}} \times \frac{3}{5}$ of $\frac{\cancel{5}}{\cancel{7}}$ of $\frac{\cancel{14}^{2}}{\cancel{19}} = \frac{6}{5} = 1\frac{1}{5}$

2. Multiply $\frac{3}{10}$ by $\frac{20}{27}$. *Result* $\frac{2}{9}$.

3. Multiply $\frac{15}{28}$ by $\frac{14}{25}$. *Result* $\frac{3}{10}$.

4. Multiply $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$. *Result* $\frac{5}{36}$.

5. Multiply $\frac{1}{9}$ of $\frac{7}{8}$ of $\frac{11}{12}$ by $\frac{6}{7}$ of $5\frac{3}{7}$. *Result* $\frac{209}{504}$.

6. What is the continued product of $\frac{15}{17}$, $\frac{34}{45}$, $\frac{9}{10}$, and $\frac{25}{27}$? *Ans.* $\frac{5}{9}$.

7. Required the continued product of $7\frac{5}{9}$, $2\frac{1}{4}$, $3\frac{1}{2}$ and $\frac{6}{7}$ of $\frac{13}{17}$? *Ans.* 39.

8. Required the continued product of $\frac{4\frac{3}{5}}{7}$, $\frac{7}{5\frac{1}{3}}$, and $\frac{9\frac{3}{4}}{11}$? *Ans.* $\frac{2691}{3520}$.

9. Required the continued product of $6\frac{1}{8}$, $2\frac{2}{3}$, $\frac{4}{7}$ of $\frac{3\frac{1}{2}}{24}$, and $\frac{3}{9\frac{4}{5}}$. *Prod.* $\frac{5}{12}$.

10. Required the approximate values of the continued product of $\frac{5}{8}$ of $\frac{3}{11}$, $\frac{7}{16}$ of $\frac{3\frac{4}{5}}{5}$, $\frac{6}{11}$ of $\frac{17}{18}$, and $\frac{13}{19}$ of $\frac{29}{37}$.

Result $\frac{1}{63}$, $\frac{1}{64}$, $\frac{7}{447}$, $\frac{8}{511}$, $\frac{15}{958}$.

DIVISION OF VULGAR FRACTIONS.

RULE.

Prepare the given fractions as in multiplication, then invert the divisor or divisors, and proceed as in that rule.

EXAMPLES.

1. Divide $4\frac{5}{9}$ by $\frac{5}{9}$ of 4.

$$\frac{4\frac{5}{9}}{9}$$

$$\frac{41}{\cancel{9}} \times \frac{\cancel{9}}{5} \text{ of } \frac{1}{4} = \frac{41}{20} = 2\frac{1}{20}$$ *Result.*

2. Divide $\frac{2}{3}$ of $\frac{4}{5}$ by $\frac{5}{7}$ of $\frac{3}{4}$ of $\frac{7}{8}$.

$\frac{2}{3}$ of $\frac{4}{5} \times \frac{7}{5}$ of $\frac{4}{3}$ of $\frac{8}{7} = \frac{256}{225} = 1\frac{31}{225}$ *Result.*

3. Divide $3\frac{4}{5}$ by $2\frac{5}{7}$. *Result* $1\frac{2}{5}$.

4. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$. *Result* $\frac{2}{3}$.

5. Divide $\frac{9}{11}$ by $\frac{3}{5}$ of $\frac{4}{7}$. *Result* $2\frac{17}{44}$.

6. Divide $\frac{1}{5}$ of 19 by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$. *Result* $9\frac{1}{2}$.

7. Divide $28\frac{8}{9}$ by $3\frac{2}{3}$. *Result* $7\frac{29}{33}$.

8. Divide $\$259\frac{7}{16}$ equally among 15 persons and what is the share of each? *Ans.* $\$17\frac{71}{[illegible]}$. 71/24

9. Divide $\frac{8}{9}$ of $\frac{5}{7}$ of $4\frac{3}{4}$ by $\frac{2}{3}$ of $\frac{4}{5}$ of $2\frac{5}{7}$. *Result* $2\frac{1}{12}$.

THE SINGLE RULE OF THREE IN VULGAR FRACTIONS.

RULE.

State the question according to the directions given in page 63. Then having reduced, when necessary, the similar terms to the same denomination, and mixed numbers to improper fractions, invert the first term, and multiply this term thus inverted and the other two continually together, for the answer sought; which answer will be of the same denomination as the third term.

Note. See the note in multiplication of vulgar fractions.

EXAMPLES.

1. If $\frac{3}{5}$ of an ounce of silver be worth $\frac{16}{23}$ of a dollar, what is the value of $5\frac{3}{4}$*oz* ?

$$\begin{array}{lccccc} & \text{oz.} & & \text{oz.} & & \$ \\ \text{As} & \frac{3}{5} & : & 5\frac{3}{4} & :: & \frac{16}{23}. \end{array}$$

$$\frac{5}{3}\times\frac{\cancel{23}}{\cancel{4}}\times\frac{\cancel{16}^{4}}{\cancel{23}}=\frac{20}{3}=6\frac{2}{3}\ \text{Ans.}$$

2. If $\frac{7}{8}$ of a yard of cloth cost \$$2\frac{5}{8}$, what will $5\frac{5}{12}$yds. cost at the same rate? Ans. \$16 25cts.

3. Bought $5\frac{1}{3}$ pieces of muslin, each containing $27\frac{3}{4}$yds. at $\frac{3}{8}$ of a dollar per yard; what was the amount?
Ans. \$55 50cts.

4. If $\frac{9}{11}$ of an hundred weight of iron cost \$$4\frac{3}{8}$ what is the price of $5\frac{14}{35}$cwt.? Ans. \$28 $87\frac{1}{2}$cts.

5. If 16 men can finish a piece of work in $16\frac{4}{9}$ days, in what time can 12 men effect it? Ans. $21\frac{25}{27}$ days.

6. What is the value of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of a pound, at the rate of $\frac{9}{16}$ of a dollar for $\frac{5}{9}$ of a lb.? Ans. \$$\frac{27}{64}$.

7. What quantity of shalloon $\frac{2}{3}$yd. wide will line $5\frac{3}{5}$yds. of cloth $1\frac{3}{4}$yd. in width? Ans. $14\frac{7}{10}$yds.

8. If a man drink daily a dram which costs $6\frac{1}{4}$cts., how much will he expend in this manner in 20 years of $365\frac{1}{4}$ days each? Ans. \$456 $56\frac{1}{4}$cts.

9. If a person take snuff once in 10 minutes and spend $\frac{3}{4}$ of a minute in the process of snuffing, adjusting the box, and blowing the nose, how many hours will be thus spent in 7 years, allowing $13\frac{4}{5}$ hours to a snuff-taking day, and $365\frac{1}{4}$ days to the year?

Ans. { $2646\frac{189}{800}$hrs., or the wakeful hours
[of more than 5 months.

10. A father dying bequeaths to one of his sons $\frac{4}{7}$ of his

estate, and to another $\frac{12}{35}$ of the remainder; upon a division the latter bequest is found to be $200 less than the former; quere the legacy of each?

Ans. The former $1400, the latter $1200.

11. If A can mow an acre of grass in $5\frac{3}{8}$hrs., and B. can mow $1\frac{3}{4}$ acres in $9\frac{1}{3}$hrs., in what time can they jointly cut $8\frac{1}{4}$ acres? Ans. $22\frac{3}{8}$hrs.

12. How many pounds of quick lime must be thrown into a well containing $64\frac{1}{2}$ cubic feet of carbonic acid gas,* to render it respirable; a cubic foot of that gas containing $\frac{5}{43}$ of a pound of carbon, and 1lb. of lime being capable of absorbing $\frac{43}{57}$ of a pound of carbon? Ans. $9\frac{31}{86}$lbs.

13. If a horse can draw $6\frac{1}{2}$ tons on a rail-road which ascends $25\frac{3}{4}$ feet in a mile, what weight would he draw if the ascent was only $15\frac{3}{8}$ feet in a mile, the force required to move a given weight being as the ascent per mile?

Ans. $10\frac{109}{123}$.

THE DOUBLE RULE OF THREE IN VULGAR FRACTIONS.

RULE.

State the question as directed page 69. Reduce the quantities as in the last rule. Invert the terms, in the first place, and multiply these terms thus inverted, and those in the second and third places continually together, for the answer sought; which will be of the same denomination as the third term.

Note.—The operation may be frequently shortened by cancelling as in multiplication.

EXAMPLES.

1. If $3\frac{1}{2}$yds. of cloth $\frac{3}{4}$yd. in width cost $$7\frac{5}{8}$, what will be the cost of $9\frac{3}{4}$yds. of $\frac{7}{8}$ wide?

---

* This is a kind of air, known by the names of fixed air, choke-damp, &c., which has caused so many fatal accidents in wells, and old vaults. Its presence may be detected by letting down into it a lighted candle, which it will extinguish. The question above given, indicates a method of rendering it innoxious.

$$\left.\begin{array}{ccc}\text{yds.}&&\text{yds.}\\ \frac{3}{4}&:&\frac{7}{8}\\ \text{yds.}&&\text{yds.}\\ 3\frac{1}{2}&:&9\frac{3}{4}\end{array}\right\}::\$7\frac{5}{8}.$$

$$\frac{\cancel{4}}{\cancel{3}}\times\frac{\cancel{2}}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{8}}\times\frac{\cancel{3}\cancel{9}}{4}\times\frac{61}{8}=\frac{793}{32}=24\frac{25}{32}.$$

2. If the wages of 5 men for $17\frac{1}{2}$ days be $\$32\frac{13}{16}$, what sum will 15 men earn in $26\frac{1}{4}$ days? Ans. $147\frac{21}{32}$.

3. If a footman travel 294 miles in $9\frac{3}{5}$ days of $12\frac{1}{2}$ hours in length, how many days of $10\frac{2}{3}$ hours will he require to travel $76\frac{2}{9}$ miles? Ans. $2\frac{1}{12}$ days.

4. If the interest of $\$257\frac{3}{4}$ for $\frac{7}{12}$ of a year be $\$7\frac{4}{5}$, how much interest will be gained by $\$773\frac{1}{4}$ in $3\frac{1}{2}$ years? Ans. $\$140\frac{2}{5}$.

5. Supposing $7\frac{1}{2}$cwt. can be carried $27\frac{7}{9}$ miles for $\$2\frac{3}{4}$, what will be the expense of carrying $37\frac{1}{2}$cwt. $7\frac{11}{27}$ miles? Ans. $\$3\frac{2}{3}$.

6. If a family of 5 persons drink $21\frac{3}{4}$ gallons of beer in $2\frac{1}{3}$ weeks, how much will 7 persons drink in $4\frac{4}{9}$ weeks? Ans. 58galls.

7. If 12 oxen eat $7\frac{3}{5}$ acres of pasture in $5\frac{1}{2}$ weeks, how many oxen will be required to eat $13\frac{2}{3}$ acres in $10\frac{15}{19}$ weeks? Ans. 11 oxen.

8. Supposing 3 compositors set $15\frac{1}{2}$ pages in $2\frac{7}{9}$ hours, how many will be required to set $69\frac{3}{4}$ pages in $6\frac{1}{4}$ hours? Ans. 6.

◆

DECIMAL FRACTIONS.

Decimal Fractions are those which have 10, 100, 1000, &c. for their denominators, but instead of expressing them as Vulgar Fractions are expressed, they are distinguished from integers by having a point prefixed, thus, $.3=\frac{3}{10}$, $.47=\frac{47}{100}$.

In the numeration of integers, reading from right to left, the local values of the figures increase in a tenfold proportion; therefore, reading from left to right, they decrease in the same proportion. Decimals, in like manner, increase or decrease in a tenfold proportion, according as they are read from right to left or from left to right; hence decimals are a continuation of the common numerical scale to fractions of a unit.

The notation and numeration of decimals, and their relation to integers, are exemplified in the following table:—

Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.	Ten millionths.	Hundred millionths.
4	9	8	7	2	4	3.	4	5	7	8	9	3	5	7

Note.—Ciphers prefixed to whole numbers or annexed to decimals, do not change their values, because, in either case, the situation of the significant figures in the scale of numbers, is not changed by the introduction of ciphers. But, ciphers annexed to whole numbers or prefixed to decimals, increase their values in the former case, and diminish them in the latter, because, being placed between the significant figures and the decimal point, they remove those figures to places higher or lower in the numerical scale.

ADDITION OF DECIMALS.

RULE.

Place the numbers with units under units, tenths under tenths, &c., then add as in whole numbers, setting the decimal point in the result directly under those in the numbers to be added.

EXAMPLES.

476.542	5847.2814	832.6483
324.769	3476.5476	276.4871
736.427	7835.9473	657.8237
523.457	4051.7186	576.4764
564.121	5513.3414	462.7374
2625.316		

1. Add together the following numbers; 437.657, 836.947, 594.865, 869,458, 742.875, and 384.762. *Result* 3866.564.

2. What is the sum of 36.578, 876.942, 32.874, 52.826, 2794.371, 242.7394, and 1764.2739? *Ans.* 5800.6043.

3. What is the sum of 876.439, 58.7523, 972.6543, 74.8365, 829.27, and 2.36474? *Ans.* 2814.31684.

4. Required the sum of $432.476, $57.24, $832.674, $272.346, $84.29, and $27.647. *Result* $1706.673.

SUBTRACTION OF DECIMALS.

RULE.

Set the less number under the greater, with units under units, tenths under tenths &c. and subtract as in whole numbers, setting the decimal point in the remainder directly under the points in the given numbers.

EXAMPLES.

426.256	183.640	356.987	832.796
134.349	84.765	128.396	247.983
291.907			

1. From 837.642 take 579.358. *Result* 258.284.

2. Required the difference in French metres, between a French toise and an English fathom, the former being 1.94904 metres and the latter 1.8282 metres. *Ans.* .12084.

3. What is the difference, in French grammes, between a pound avoirdupois and a pound troy; the pound avoirdupois being 453.25 grammes, and the pound troy 372.96 grammes? *Ans.* 80.29 *gram.*

4. Chimboraza, the highest peak of the Andes, is calculated to be 3.97727 miles in height, and Mount Blanc the highest of the Alps, 2.96628 miles; how much higher is the former than the latter? *Ans.* 1.01099 *miles.*

MULTIPLICATION OF DECIMALS.

RULE.

Multiply as in whole numbers, and point off, in the product, as many decimals as there are in both factors.

Note.—If the product does not contain as many figures as are required to be pointed off, prefix one or more ciphers to supply the deficiency.

EXAMPLES.

Multiply 4765.2364
by 22515.375

142957092
714785460
3573927300
10721781900

107291084.4096500

	Multiply	by	Product.
2.	572.487	27.381	15675.266547.
3.	31.415926	324.819	10204.489667394.
4.	57.29578	82.472	4725.29756816.
5.	5.235988	108.273	566.916128724.
6.	.147623	.023486	.003467073778.

Note.—When the decimals in the factors are numerous, it is sometimes desirable to obtain the product to fewer places than the rule above given will produce. This may be effected in the following manner.

Place the units of the multiplier, under such figure of the multiplicand as is intended to be the lowest retained in the product, and set down the remaining part of that factor in such manner that the figures of the multiplier shall stand in an inverted order; then in multiplying, begin with the figure of the multiplicand which stands over the multiplying figure, adding, however, to this product, one for the first 5, and one for every subsequent 10, contained in the product of the next right hand figure of the multiplicand, and set the first or right hand figures of all these products under each other. Then add as before for the answer.

EXAMPLES.

7. Multiply 476.2457 by 37.2765 retaining two decimals in the product.

$$\begin{array}{rl} 476.2457 & \\ 5672.73 & \\ \hline 1428737 & \\ 333372 & \\ 9525 & \\ 3333 & \\ 286 & \\ 24 & \\ \hline 17752.77 & \textit{Result.} \\ \hline \end{array}$$

8. Multiply 724.36942 by 25.743, retaining three decimals in the product. *Result* 18647.440.

9. Multiply 573.47694 by 3.141592, reserving four decimals in the product. *Result* 1801.6305.

10. Multiply 524.83674 by 36.9472, retaining three decimals in the product. *Result* 19391.246.

11. The French toise contains 1.94904 metres, and the metre 3.28192 English feet; quere the number of English feet in a French toise, to five decimal places?

Ans. 6.39659.

12. According to the observations of Humboldt and others, the region of perpetual snow commences, under the equator, at the height of 4796 metres above the level of the sea; what is this height in yards, to the nearest integer; the metre being 1.09397*yds.* nearly?

Ans. 5250*yds.* or nearly 3*miles.*

DIVISION OF DECIMALS.

RULE.

Divide as in whole numbers, and point off in the quotient as many decimals as the decimals in the dividend exceed those in the divisor.

If the quotient does not contain as many figures as are required to be pointed off, prefix one or more ciphers to supply the deficiency.

When the dividend has fewer decimals than the divisor, annex ciphers to the former to make up the number. When remainders occur, the division may be continued at pleasure, by annexing ciphers to the dividend.

Note.—The situation of the decimal point, in the quotient, may also be determined thus—

Set the divisor under the first left hand figures of the dividend which contain it, then the first figure in the quotient will possess the same place in the numerical scale as the figure of the dividend which stands over the units of the divisor.

EXAMPLES.

1. Divide 278.364 by 59.3876.

59.3876)278.3640(4.68724
2375504
4081360
3563256
5181040
&c.
65776 rem.

Here the dividend, including the ciphers annexed, contains 9 decimals, and the divisor 4, therefore we point off 5 decimals in the quotient. Or,

Setting the divisor under the first figures of the dividend which contain it, thus,

278.3640
59.3876

we see that the units of the dividend stand over the units of the divisor, consequently the first quotient figure must be units, and all the rest decimals.

2. Divide 8642.369852 by 38.2576. *Quot.* 225.899+

3. Divide 566.916128724 by 108.273.
Quot. 5.235988.

4. Divide 10204.489667394 by 324.819.
Quot. 31.415926.

5. Divide 732.49367 by 24.73642. *Quot.* 29.61197+

6. Divide 7 by .7 *Quot.* 10.

7. Divide .376 by 3.76 *Quot.* .1.

When there are many figures in the divisor, the operations may frequently be contracted, and the result obtained with the necessary degree of accuracy.

Find, by the second method, how many places of integers the quotient will contain; whence if we desire to retain a given number of decimals, the whole number of quotient figures will be known. Then counting off as many left hand figures of the divisor as are intended to be retained in the quotient, proceed to find the quotient figures in the usual way; but in multiplying by the first quotient figure, set down the product of those figures only which are thus counted off, in the divisor, carrying to the first product as in contracted multiplication, and for each subsequent figure in the quotient reject one figure from the right hand of the divisor.

8. Divide 7884.367597 by 36.4876467 retaining 5 decimals in the quotient.

```
36.4876467)7884.367597(216.08320 quotient.
 ' ' ' ' ' ' ' '      72975293
                      --------
                       5868382
                       3648765
                       -------
                       2219617
                       2189258
                       -------
                         30359
                         29190
                         -----
                          1169
                          1094
                          ----
                            75
                            73
                            --
                             2
```

9. Divide 264.3789672 by 38.476536 reserving 6 decimals in the quotient. Quotient 6.871174.

10. Divide .69314718 by 2.302585 retaining 5 decimals in the quotient. Quotient .30103.

11. Divide 4109.2351 by 230.409 so that the quotient may contain only 4 decimals. Quot. 17.8345.

12. Divide 846.76954 by 37.246987 retaining 6 decimals in the quotient. Quot. 22.733907.

13. Divide 243.64753 by 79.38437 retaining 5 decimals in the quotient. Quot. 3.0692.

REDUCTION OF DECIMALS.

CASE 1.

To reduce a vulgar fraction to its equivalent decimal.

RULE.

Divide the numerator, with ciphers annexed, by the denominator, observing that these ciphers must be considered as decimals; the quotient, pointed as directed in the preceding rule, will be the decimal required.

Note.—When the given fraction is a compound one, it must be reduced to a simple fraction.

EXAMPLES.

1. Reduce $\frac{7}{16}$ to its equivalent decimal.

16)7.0000

.4375 Result.

2. Reduce $\frac{3}{4}$ to a decimal. Result .75

3. What decimal is equivalent to $\frac{5}{26}$? Ans. .1923076+

4. What is the value of $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{5}{16}$ in decimals? Ans. .205078125.

5. Reduce $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{4}{9}$ to an equivalent decimal. Result .238095+

6. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ to an equivalent decimal. Result .5.

7. Beduce $\frac{7}{8}$ of $\frac{13}{14}$ of $\frac{19}{26}$ to an equivalent decimal. Result .59375.

8. Reduce $\frac{8}{9}$ of $\frac{3}{5}$ of $\frac{11}{12}$ to an equivalent decimal.
Result .488888+

CASE 2.

To reduce a given quantity to the decimal of an integer of a higher denomination.

RULE.

Reduce the given quantity to the fraction of an integer of the denomination proposed, by case 8, Reduction of Vulgar Fractions, and reduce this fraction to a decimal by the last case.

Note.—When the given quantity consists of divers denominations, it is most convenient to reduce the terms successively, beginning with the lowest denomination, annexing the decimal in each case to the integer of the same denomination.

EXAMPLES.

1. Reduce 17s. 6d. 3qrs. to decimals of a £.

s. d. qrs.
17 6 3
12

210
4

$\frac{843}{4}$ of $\frac{1}{12}$ of $\overset{£.}{\frac{1}{20}} = \overset{£.}{\frac{843}{960}} = \overset{£.}{\frac{281}{320}}$)281.0(.878125£. Result.
256

250
224

260
256

4 &c.

Or thus,

4	3.*qrs.*
12	6.75*d.*
20	17.5625*s.*
	.878125£. the same as before.

2. Reduce 6oz. 15dwts. 18grs. to decimals of a pound troy. Result .565625lb.

3. Reduce 7s. 6d. to decimals of a pound. Result .375£.

4. Reduce 3qrs. 2na. to decimals of a yard. Result .875 yd.

5. Reduce 3qrs. 14lb. to decimals of a Cwt. Result .875 Cwt.

6. Reduce 1yd. to the decimal of an ell English. Result .8 E.E.

7. Reduce 5Cwt. 3qrs. 16lb. to decimals of a ton. Result .2946428+ton.

8. Reduce 3 pecks 6qts. 1pt. to decimals of a bushel. Result .953125bu.

9. Reduce 14hrs. 45min. 36sec. to decimals of a day.
34 60 Result .615 days.

10. Reduce 4sin. 20° 35′ to decimals of the whole circumference. Result .390509+

CASE 3.

To reduce decimals to their proper values.

RULE.

Multiply the decimals as in common Reduction, pointing off as many decimals in the product as are contained in the multiplicand. The integers thus obtained will be the values required.

EXAMPLES.

1. Reduce .7875lb. troy to its proper value.

```
          lb.
        .7875
           12
       ------
   oz. 9.4500
          20
       ------
dwts. 9.00        Result 9oz. 9dwts.
```

2. What is the value of .878125£. sterling?
Ans. 17s. 6d. 3qrs.
3. What is the value of .875Cwt. Ans. 3qrs. 14lb.
4. Reduce .67457lb. avoirdupois to its proper value.
Result 10oz. 12.68992drs.
5. What is the value of .8435yd?
Ans. 3qrs. 1.496na.
6. What is the value of .9675mile?
Ans. 7fur. 29per. 3 yds. 10.8sin.
7. What is the value of .63478 of 25 acres?
Ans. 15ac. 3r. 19.12per.
8. Reduce .5675 day to its proper value.
Result 13h. 37min. 12sec.
9. What is the value of .576 of 3C. 2qr?
Ans. 2C. 1lb. 12oz.10,752drs.
10. Reduce .7346hhd. to its proper value.
Result 46galls. 1.1192qts.
11. Reduce .859375 bushel to its proper value.
Result 3p. 3qts. 1pt.

THE SINGLE RULE OF THREE IN DECIMALS.

RULE.

State and work the questions agreeably to the directions given in whole numbers; having due regard to the position of the decimal points.

EXAMPLES.

1. If 1.5cwt. of sugar cost \$11.76 what is the value of 16C. 3qrs. 14lb?

```
                     C.       C.          $
28|14.          As 1.5  :  16.875  ::  11.76
 4| 3.5                    11.76
   16.875                 -------
                   1.5)198.45000
                          -------
                  Ans. $132.3=$132 30cts.*
```

---

* As the denominations of federal money are graduated according to a decimal scale, it is evident that any one of its denominations may be assumed as units, and those of a lower order as deci-

2. If 19yds. of linen cost $14.25 what will 435.6yds. amount to ? Ans. $326.70.

3. What is the price of 5C. 3.6qrs. of iron at the rate of $2.75 per cwt ? Ans. $16.225.

4. The value of 8.25lb. troy of pure silver, being $128, what are 376.7848lbs. troy worth ? Ans. $5845.873+

5. If 576.75yds. of cloth be purchased at $2.5625 per yard, and sold for $2.875 per yard, quere the gain ? Ans. $180.234375+

6. By a set of experiments, recently made in France, sound has been found to move 1106.3 feet per second; what is the distance of a cloud in which the lightning was observed 47.5 seconds before the thunder was heard? Ans. 9.952+miles.

7. When the mercury in the barometer stands at 2.5 feet, required the height of a column of sea water which would be supported by the same atmospheric pressure, supposing the specific gravity of mercury to that of sea water as 13.6 to 1.03 ? Ans. 33.009+feet.

8. If 36.75yds. of cloth of 1.25yds. in width be exchanged for a quantity only .75yds. wide, how many yards are equivalent, the quality being the same ? Ans. 61.25yds.

9. If $783.42 gain $37.45 in 12.75 months, what principal will gain the same interest in 15.3 months ? Ans. $652.85.

10. If by travelling 12.3 hours in a day, a jonrney be performed in 24.55 days, in how many days will it be accomplished by travelling 10.25hrs. in a day ? Ans. 29.46 days.

11. If a quantity of water, which at the temperature of 40 degrees would measure 65.5 gallons, be heated to the boiling point; quere the number of gallons; the volumes of water at these temperatures being in the ratio of 1 to 1.04785 ? Ans. 68.634175galls.

12. It has been found that 100 cubic inches of alcohol, and 82.5 of water, each at the temperature of 60 degrees,

---

mals; it is usual, however, to reckon the dollars as units and dimes, cents, &c. as tenths, hundredths, &c. It is also manifest that whether a sum, in this currency, be expressed in all its different denominations, or in decimals of one only, the figures used are precisely the same.

when mingled and reduced to the same temperature, measure only 177.41 cubic inches; if therefore 125galls. of the former and 103.125galls. of the latter, be mingled, under these circumstances, how many gallons will the mixture compose? Ans. 221.7625galls.

13. The circumference of the earth, measured on the equator, being 24896.16 miles, what is the length of an arc of 57° 17′ 45″? Ans. 3960.3505 miles.

14. The American eagle contains 247.5 grains of pure gold, and 64 eagles are equivalent to 133.5 English guineas, how many tons of pure gold would be required to pay the British national debt, of 863751990 guineas?
Ans. 22387t. 3c. 1q. 22lb.

THE DOUBLE RULE OF THREE IN DECIMALS.

RULE.

Proceed as in whole numbers, observing the directions heretofore given, with regard to the position of the decimal points.

EXAMPLES.

1. If the interest of $76.48 for 1.25yrs be $5.736, what will be the interest of 2576.8 for 2.75yrs?

$$\left.\begin{array}{rcr}
\$. & & \$. \\
{}^{8}\cancel{76.48} & : & {}^{8}\cancel{2576.8} \\
{}^{4}\cancel{9.56} & & 322.1 \\
2.39 & & \\
\text{yrs.} & & \text{yrs.} \\
{}^{.25}\cancel{1.25} & : & {}^{.25}\cancel{2.75} \\
5 & & 11 \\ \cline{1-1}
11.95 & &
\end{array}\right\} :: \begin{array}{r}
\$. \\
{}^{4}\cancel{5.736} \\
1.434 \\
11 \\ \hline
15.774 \\
322.1 \\ \hline
\end{array}$$

Ans. 11.95)5080.8054($425.172.

2. If $4.56 be paid for the carriage of 10.25Cwt 20 miles, how much must be paid for the carriage of 15.375 Cwt 7.5 miles? Ans $2.565.

3. If 5 men in 6 days mow 52.2 acres of grass, how many men must be employed to mow 417.6 acres in 12 days? Ans. 20 men.

4. If a courier travelling 14.785 hours in a day, perform a journey of 825.36 miles in 976 days; how many days of 11.828 hours will he require to travel 1238.04 miles? Ans. 18.3 days.

5. If a piece of cloth 28.75yds long, and 1.5yds wide, cost $129.375, what will be the price of another piece of the same quality, 43.125yds. in length, and 1.2yds in breadth? Ans. $155.25.

6. If a cellar 22.5ft. long, 17.3ft. wide, and 10·25ft. deep, be dug in 2.5 days by 6 men, working 12.3 hours in a day; how many days of 8.2 hours, would 9 men require to dig another, which will measure 56.25ft. long, 25.95 feet wide, and 16.4 feet deep? Ans. 15 days.

7. If 18 men consume $$78\frac{3}{4}$* worth of flour in $9\frac{1}{2}$ months, when wheat is sold at $$2\frac{1}{8}$ per bushel, in how many months will 35 men consume $$145\frac{7}{8}$ worth when wheat costs $$1\frac{7}{8}$ per bushel? Ans. 10.256+

8. If 24 men, in $180\frac{7}{8}$ days of $14\frac{1}{2}$ hrs. in length, dig a trench $33\frac{3}{4}$ yds. long, $3\frac{1}{2}$ deep and $5\frac{3}{5}$ wide, how many days of $15\frac{4}{5}$ hrs. will 217 men require to excavate one of $23\frac{1}{4}$ yds. long, $2\frac{1}{4}$ deep and $3\frac{7}{8}$ wide? Ans. 5.6257+

FELLOWSHIP.

Fellowship is the application of the rules of proportion to the adjustment of the gain or loss, on any joint adventure, among the several parties concerned, or to the division of the property of bankrupts among the creditors.

---

*These numbers are given in the form of vulgar fractions, to exercise the pupil in the reduction of such fractions into decimals.

CASE 1.

Sometimes called Single Fellowship.

When the stocks in company are considered without regard to time.

RULE.

As the amount of stock or the whole debt
Is to any partner's share thereof,
So is the whole gain, loss, or property to be divided,
To that partner's share or dividend ;

Or, Find the dividend corresponding to one dollar or one unit of the given denomination of stock, either in decimals or vulgar fractions ; and multiply each partner's stock or claim by this fraction for the dividend required.

PROOF.

Add the several dividends together, the amount will be equal to the whole sum to be divided.

EXAMPLES.

1. The money and effects of a bankrupt amount to $2576 70 cts. and he is indebted to A. $1956 75 cts. to B. $2372 50 cts. and to C. $1396 75 cts ; required the several shares or dividends ?

$
1956.75
2372.50
1396.75
―――
5726.00

	$.		$.		$.		$.	
As	5726	:	1956.75	::	2576.70	:	880.5375	A.'s share.
	5726	:	2372.5	::	2576.70	:	1067.625	B.'s share.
	5726	:	1396.75	::	2576.70	:	628.5375	C.'s share.
							2576.70	proof.

Or thus,

```
5726.)2576.7(.45 per $
      22904
      ─────
       28630
       28630
       ─────
```

1956.75×.45= 880.5375 A.'s share.
2372.5 ×.45=1067.625 B.'s share.
1396.75×.45= 628.5375 C.'s share.

2. Three partners D. E. and F. trading together gained 120 dollars; D.'s stock was $280, E.'s $600, and F.'s $320; required the share of each.

Ans. D. $28, E. $60, F. $32.

3. A. B. C. and D. freighted a ship with wine, of which A. had 25 tuns, B. 36, C. 40, and D. 43; but the crew threw overboard, in a storm, 36 tuns; what part of the loss must they severally bear?

Ans. A. 6.25 tuns, B. 9, C. 10, and D. 10.75.

4. A legacy of $3050 25cts. is to be divided among A. B. and C. in proportion to the number of their children, now A. has 6, B. 7, and C. 8; required their respective shares. Ans. A. $871.50; B. $1016.75; C. $1162.

5. A person failing in trade is indebted to A. $539, to B. $756 80cts. to C. $854 16cts. and to D. $1200; his property, when disposed of, amounts to $637 49cts; how much will they severally lose?

Ans. A. $404 25cts. B. $567 60cts.
C. $640 62cts. D. $900.

6. D. E. and F. join in trade, with stocks in the ratio of the numbers of 2, 3, and 4, respectively; the whole amount of their stock is $1215 and their gain equal to D.'s stock; required the stock and gain of each.

	$		$
Ans. D.'s stock	270,	gain	60.
E.'s stock	405,	gain	90.
F.'s stock	540,	gain	120.

7. A gentleman meeting, by accident, with a dangerous wound, sends for three physicians, promising to divide $100 among them, in the ratio of the reciprocals of the number

of minutes which shall elapse before they attend. The first arrives in 25 minutes, the second in 30, and the third in 35; quere their respective shares.

Ans. the 1st $39 25cts. 2d $32 71cts. 3d $28 04cts.

CASE 2.

Sometimes called Double Fellowship.

When the stocks are in trade during different times, or increased or diminished while the partnership continues.

RULE.*

Multiply the several stocks by the times they were respectively in trade. Then

As the sum of the products
Is to the product of each stock and time,
So is the whole gain, loss, or sum to to divided,
To each partner's share or dividend.

Or, divide the whole gain, loss, &c. by the sum of the products, and multiply the particular products by the result.

Note.—This rule with a little modification, is easily applied to cases in which the several stocks and times, and the gain of one partner, are given to find the rest; the several times and gains, and one of the stocks to find the rest, &c. The principle of the solution being, that the several gains are to each other in the ratio of the stocks multiplied by their respective times.

EXAMPLES.

1. A. B. and C. join stocks for 12 months; A. puts in $100, and on the first of the 5th month $150 more; and on the 1st of 9th month takes out $30. B. puts in at first $250, on the first of 6th month $60 more, and on the 1st of 11th month $100 more. C. puts in $300, on the 1st of the 4th month takes out $200, and on the 1st of the 8th month takes out $50 more. At the end of the year their gain is found to be $149; required their several shares.

---

* The times must all be taken in the same denomination; and they may then be considered as abstract numbers.

100×4= 400
250×4=1000
2204×= 880
A's stock×his time 2280

250×5=1250
310×5=1550
410×2= 820
B's stock×his time 3620

300×3=900
100×4=400
50×5=250
C's stock×his time 1550

2280
3620
1550
7450

			$.	$. cts.	
As 7450	: 2280	: :	149 :	45 60	A's gain.
7450	: 3620	: :	149 :	72 40	B's gain.
7450	: 1550	: :	149 :	31 00	C's gain.
				149 00 proof.	

Or thus,
7450)149.00(.02
149.00

$
2280×.02=45.60 A's gain.
3620×.02=72.40 B's gain.
1550×.02=31.00 C's gain.

2. Three graziers, M. N. and P. rent a tract of pasture in common, for $120, into which M. puts 80 oxen for 35 days, N. 75 oxen 50 days, and P. 90 oxen 45 days; what rent must they severally pay?

Ans. M. $$31\frac{37}{53}$, N. $$42\frac{24}{53}$, P. $$45\frac{45}{53}$.

3. A. and B. commence trade in company; A. contributes at first $500, and B. $750, at the end of 4 months A. adds $600 to his stock; what sum must B. advance at the same time, to entitle him, at the end of the year, to equal profits with A.? *Ans.* $225.

4. A. B. and C. made a stock for 12 months; A. put in at first $364, and 4 months afterwards he put in $40 more; B. put in at first $408, and at the end of 7 months he took out $86; C. put in at first $148, and 3 months afterwards added $86, and 5 months after that he put in $100 more; at the end of 12 months their gain is found to be $1436; what is each man's share thereof?

Ans. A. $556 18cts., B. $529 84cts. C. $349 98cts.

5. A. B. and C. form a joint stock of $4392, by which they gain $234: A's money is in trade 4 months, B's 5 months, and C's 13 months; and their shares of the gain are as the numbers 2, 3, and 4, respectively. Required the stock and gain of each.

$. $.
Ans. A's stock 1560, gain 52.
B's stock 1872, gain 78.
C's stock 960, gain 104.

EXCHANGE.

The business of Exchange is to reduce the moneys of one country or government to those of another.

The denominations of money are either real or imaginary.

They are real when the terms designate particular coins, as Eagles, Dollars, Guineas, Crowns, Napoleons, &c.

They are imaginary when the terms denote some supposed divisions or aggregates of money, not actually coined, as the pound of Great Britain, the livre tournois of France, &c.

The same term, for imaginary sums, is often used at different times or in different places, to denote quantities of various intrinsic* values: thus, under the reign of William II. (1087,) the English money pound contained one tower pound of standard silver, or 11oz. 2dwts. of pure silver, and 18dwts. of alloy; but at present the pound troy of silver of equal purity, is reckoned at 3.3 pounds sterling.

---

* The intrinsic value of any coin or sum, is estimated by the quantity of pure silver or gold which it contains. Gold and silver coins always contain a portion of baser metal, chiefly copper, called alloy, the proportion of which to the pure metal is regulated by the government which authorizes the coinage. The use of alloy is to render coins less subject to abrasion. The relative values of gold and silver are in great measure arbitrary, being generally settled by the authority of government, and are seldom alike in different countries. In the United States, pure gold is to pure silver as 15 to 1.

The tower pound which was used in the English mint until the time of Henry VIII. is equal 11¼oz. troy. At this time the pound sterling, the pound Irish, and the pound of the British West Indies, are of different intrinsic values.

In some places, particularly Venice, and some cities in the Netherlands, where banks of deposit have been long established, payments are frequently made by transfers of the credits, on the books of the banks, from one person to another. The sums deposited in these banks are termed banco, and being free from the casualties incident to the common circulating medium, usually bear a premium, which is called the agio. The agio is computed at so much per cent.

When remittances are made in cash, exchanges may be readily computed from the intrinsic or legal values of the coins remitted: but as they are generally made, between distant places, by bills of exchange, and those bills seldom pass according to the intrinsic values of the sums for which they are drawn, the rate of exchange must be taken into the account.

Whenever, in the commercial transactions between distant places, the debits and credits are nearly equal, bills of exchange may be considered as negotiable according to the intrinsic values of their nominal amounts: exchanges are then said to be at par. But when the debits become greater in amount than the credits, exchanges must rise, or sell above par.

DOMESTIC EXCHANGE.

Under the colonial governments, the commercial transactions between the colonies were loaded with the embarrassments resulting from the use of imaginary sums, and the expression of different quantities of money by the same term: but the system established by the federal government has given complete uniformity to the money of all parts of the United States. It therefore appears unnecessary to treat any further on this branch of exchange than to show the manner of reducing the ancient currencies to that now in use.

The old currencies of the several states, compared with the present, are,

Federal money*	New England and Virginia.	New York and North Carolina	Penn. N. Jersey, Delaware and Maryland	S. Carolina and Georgia.
1 dollar.	6 shillings.	8 shillings.	7*s*. 6*d*.	4*s*. 8*d*.

Hence any sum, in either of these currencies, may be easily reduced to federal money, by simple proportion, or more concisely by the following

RULE.

Reduce the given sum into pounds and decimals of a pound, then

For New England or Virginia, multiply by 10 and divide by 3.

New York and North Carolina, multiply by 10 and divide by 4.

Pennsylvania, New Jersey, Delaware and Maryland, multiply by 8 and divide by 3.

South Carolina and Georgia, multiply by 30 and divide by 7.

The result will be the value in dollars and decimals of a dollar.

EXAMPLES.

1. An old bond amounting to £703 15s. 7½d., Pennsylvania currency, being presented for payment, required its value in federal money.

---

* The dollar of the United States contains 17 *dwts.* 8 *grs.* of standard silver, or 371.25 *grs.* of pure silver and 44.75 *grs.* of alloy. The Eagle 11 *dwts.* 6 *grs.* of standard gold, or 247.5 *grs.* of pure gold and 22.5 *grs.* of alloy.

```
 2| 1.
12| 7.5
20|15.625
  |________
  £703.78125
           8
  __________
3)5630.25
  _______
 $1876.75 Ans.
```

2. The land in Pennsylvania was commonly sold, in the beginning of last century, at 4 shillings an acre; required the price in federal money, of 4758 acres at that rate. $2537 60cts.

3. Required the value of £5652 16s. 6d. New England currency in federal money. $18842.75.

4. An old debt of £8654 17s, 9d. in the state of New York being now about to be paid; required the amount in federal money. Ans. $21637.21875.

5. An old demand of £1952 16s. Georgia currency being recently recovered, required the amount in federal money. Ans. $8369.14+

6. Dr. Franklin's legacy to the city of Boston was £1333 6s. 8d. New England currency; what was its value in federal money? Ans. $4444.44.

FOREIGN EXCHANGE.

In the annexed tables, the denominations of money are given as they are reckoned in the countries mentioned; and the values of the coins are calculated according to the quantity of pure silver or gold which they contain, compared with the coins of the United States. To understand these comparisons, the student should attend to the remarks at the end of the note page 122.

GREAT BRITAIN.

4 farthings, (qrs.) = 1 penny, d.
12 pence . . . = 1 shilling, s.
5 shillings . . = 1 crown, (silver) = (coined before 1816) $1.1573—(since 1816) $1.0872.
20 shillings . . = 1 pound, £
21 shillings . . = 1 guinea, (gold) = $4.7940+

The pound troy of standard gold, or 11oz. of pure gold and 1oz. of alloy, is coined into 44½ guineas. Hence 44½ guineas are equal in weight and value to 21⅓ federal eagles. The pound troy of English standard silver contains 11oz. 2dwts. of pure silver, and 18dwts. of alloy, and was coined, from 1601 till 1816, into 62 shillings. During that time 222 grs. of pure silver were valued at 31 pence sterling,* but in 1816, there was a new coinage at the British mint, making 66 shillings of the pound troy of standard silver, instead of 62. Consequently 74 grs. troy of pure silver are now equivalent to 11d. sterling. The new gold coin, called a sovereign, is of the value of 1 pound sterling = $4.5657. The relative values of gold and silver to England, are as 14.2878099 to 1.

SPAIN.

34 maravedies { vellon of plate. } = 1 rial { vellon of plate. }
8 rials of plate . . . . . = 1 piaster of exchange.
10 rials of plate . . . . . = 1 dollar, (silver,)†

---

* The sum sterling generally denotes the value as reckoned in Great Britain itself. In the time of king John, (about 1210,) money coined in the eastern parts of Germany came into special request in England, on account of its purity, and was called easterling money; the inhabitants of those countries being then called easterlings. Soon after, some of those people skilled in coining, were sent for to perfect the English coins, which coins were thence termed sterling, from easterling.

† According to assays made at the United States' mint, the average value of the Spanish dollar is 1.0026, the French crown or ecu $1 10, and the five francs piece $.9323, differing, in the third deci-

375 maravedies of plate . . =1 ducat of exchange.
36 rials of plate . . . =1 pistole, (gold)= $3.7907 since 1772
32 rials vellon are worth 17 rials of plate.

FRANCE.

Old System.—12 deniers . . =1 sol or sou,
20 sous . . =1 livre tournois,
6 livres tour. =1 crown (silver)= $1.1024.
24 livres tour. =1 Louis d'or, (gold)=$4.2836.
New System.—10 centimes . =1 decime,
10 decimes . =1 franc,
40 francs . =1 Napoleon, (gold) =$7.2456.

The 5 francs piece weighs 25 grammes, or 386.4grs. troy, nine parts of which in ten are pure silver, consequently 5 francs=$.9362 United States money. Also, 80 francs =81 livres tournois.

PORTUGAL.

20 reas . . . . . =1 vintin,
5 vintins or 100 reas=1 testoon,
4 testoons . . . =1 old crusade,
24 vintons or 480 reas=1 new crusade, silver= $.6441. gold= $.5802.

---

mal, from the value as found by a comparison of their standard weights with those of the United States' coins. The Spanish dollars, coined about the beginning of the last century, contained 387 grs. of pure silver. From this standard they successively fell off in 1731, in 1761, and 1772. In the last of these years the legal weight appears to have been fixed at 420grs. of which 10¾ parts in 12 were to be pure silver; but upon taking the average of a number of those dollars in actual circulation, they were found to weigh 416grs; of which 371¼grs. were pure silver. Hence the American congress in 1792 settled the weight and purity of the federal dollar. This dollar is worth about 4s. 7d. of the present sterling silver coins.

1000 reas . . . . . =1 milrea,
48 testoons . . . =1 moidore, (gold)=$6.1637.
64 testoons . . . =1 Johannes, (gold)=$8.2183.

HOLLAND.

8 pennings . . . . =1 groot,
2 groots or 16 pennings=1 stiver,
12 groots or 6 stivers =1 shilling,
20 stivers . . . . . =1 guilder or florin,
2½ guilders. . . } 1 Albert's dollar, (silver,)= $1.0145.
52 stivers . . . } =1 rix dollar banco, (silver,) =$1.0550.
20 shillings . . . . . 1 pound flemish.

HAMBURG.*

12 pfenings . . . . . . =1 schilling lubs,
16 schilling lubs . . . =1 mark,
3 marks . . . . =1 rix dollar, silver=.9175
6 pfenings . . . . . . =1 flemish penny,
12 pence or 6 schillings lubs=1 shilling flemish,
20 shillings flemish, or 2½ rix dollars, } . =1 pound flemish.

Banco bears an agio of 19 or 20 per cent.

The moneys of Norway and Denmark are sometimes reckoned as in Hamburg; but the rix dollars of Denmark are of various kinds and values; the highest being equal to $1.0556 U. S.

SWEDEN.

12 runstucks . =1 schilling,
48 schillings . =1 rix dollar, silver=$1.0687.
2 rix dollars . =1 ducat, gold =$2.1393.

---

* There appears to be a great variety of coins in circulation in this city, the relative values of which are not settled by any proper standard. Oddy, in his Treatise on Commerce, relates, that an Englishman, by way of experiment, took a guinea to change from one coin to another, till he reduced it to eight pence in a very few hours, when he discontinued the experiment.

They have copper coins and silver ones, bearing the same names, the current values of the latter being 3 times those of the former.

RUSSIA.

2 denuscas . . . . =1 copec.
50 copecs . . . . . =1 poltin.
2 poltins or 100 copecs =1 rouble, silver, (coined 1801)=$.6479.
10 roubles . . . . . =1 imperial, gold,=7.4881.

The rouble of 1764, now out of circulation, contained about 37 grs. of silver more than the rouble of 1801.

AUSTRIAN DOMINIONS.

5 pfenings . =1 creutzer.
60 creutzers . =1 florin.
2 florins . . =1 dollar of 1753, silver, =$.9713.
4¼ florins . . =1 Hungarian ducat, gold,=$2.1531
6⅓ florins . . =1 soverain, gold=$3.1664.

PRUSSIA AND POLAND.

18 pfenings . . . . . =1 grosche.
30 groschen . . . . . =1 florin.
3 florins . . . . . . =1 rix doll. silver=$.6937.
5 rix dollars 8 groschen =1 Fredrick d'or, gold=$3.7725.

GENEVA.

12 deniers . . =1 sou.
20 sous . . . =1 livre.
3 livres . . . =1 ecu, silver,=$.9357.
10 livres . . . =1 pistole, gold,=$3.22505.

GENOA.

12 denari =1 soldo.
20 soldi . =1 lire.
10 lire . =1 scudo d'argento, silver,=$1.5347.
13½ lire . =1 sequin, gold, =$2.1664.
100 lire . =1 genovina d'oro, gold,=$16.0299.

NAPLES.

10 grains . . . = 1 carlin.
10 carlini . . . = 1 ducat, silver, =$.8265.
6 ducats . . . = 1 double ounce, gold. = $4.8004.

ROME.

5 quatrini . . . = 1 bajocco.
100 bajocchi . . = 1 scudo, or crown, silver, = $1.0204.
152.3 bajocchi . . . = 1 gold crown.
12 golden denari . = 1 golden soldo.
20 golden soldi . . = 1 golden crown.

VENICE.

12 denari . . = 11 soldo.
20 soldi . . . = 1 lira.
6 lire 4 soldi . = 1 ducat of account.
8 lire . . . = 1 ducat effective, silver, = $.7839.
22 lire . . . = 1 sequin, gold = $2.1705.

The annexed table contains the principal gold and silver coins now current, not included in the preceding, with their intrinsic values in money of the United States.

		$
Bavaria,	Caroline d'or, gold	4.7345
	Max d'or	3.1564
Basil,	Rix dollar, coined since 1764, silver	.8191
Bern,	Ducat, gold	2.1487
	Patacon, silver	.9776
	10 Batres pieces, do	.2851
Brunswick,	Carl d'or, gold	3.7479
East Indies,	Mohur, or gold rupee	6.8343
	Star pagoda, gold	1.7317
	Rupee Sicca, silver	.4740
	Do. Bombay and Madras, do.	.4700
	Do. Surat, do.	.4435
	Do. Arcot, do.	.4588
Flanders,	Ducatoon, do.	1.2025
Hanover,	George d'or, gold	3.7725
	Gulden, do.	1.5184
Holland,	Ryder, do.	5.6869
	Ducat, do.	2.1378
	Ducatoon, silver	1.2732

		$
Malta,	Louis d'or, double gold	8.9798
Milan,	Doppia or pistole, gold	3.5680
	Fillipp, silver	1.0939
Piedmont,	New Pistole, (coined since 1785) gold,	5.1559
	Sequin do.	2.1503
	Scudo, silver	1.3257
Saxony,	Auguste d'or, gold	3.7103
Sicily,	Ounce, do.	2.4380
	Ounce, silver do.	2.5119
	Scudo, do.	1.0048
Turkey,	Sequin of 1764, gold	2.0966
	Mahub of 1781, do.	1.3160
	Nisfie of 1781, do.	.6580
	Piastre of 1780, silver	.3730
	Do. of 1801, do.	.2578
Tuscany,	Ruspono, gold	6.4933
	Sequin gigliato, do.	2.1644
Zurich,	Ecu, or rix dollar, silver	1.0150

Note.—Foreign coins are not now a legal tender in the payment of debts. In calculating duties, *ad valorem*, on invoices given in foreign currencies, the following rates are settled by the laws of the United States.

	$
The Pound sterling to be estimated at	4.44
Livre tournois, of France	.185
Florin or guilder of United Netherlands	.40
Mark banco of Hamburg	33⅓
Rix dollar of Denmark	1.00
Rial of plate of Spain	.10
Rial vellon of do.	.05
Milrea of Portugal	1.24
Pound of Ireland	4.10
Tale of China	1.48
Pagoda of India	1.94
Rupee of Bengal	.555

And all other denominations of money as nearly as may be to these rates, or to their intrinsic values compared with money of the United States.

RULE.

Work by the Rule of Three or by Practice or Decimals as may be most convenient.

EXAMPLES.

1. A merchant in Philadelphia imports goods from London amounting, per invoice, to £1525 16s. 9d. sterling: what sum, in money of the United States, must be given for a bill of exchange to that amount, exchange being at $4.75 per £ sterling?

£1525 16s. 9d.=£1525.8375

$ $ cts.

1525.8375×4.75=7247.728125 or 7247.73 nearly.

2. An English emigrant has brought with him 550 English guineas, which he wishes to exchange for money of the United States, what sum is equivalent thereto?

Ans. $2636.70.

3. A merchant in Boston exports goods to France, which are sold there for 7576 francs; what is the value of a draft on his factor for that amount, at 18cts. per franc?

Ans. $1363.68cts.

4. Received from Lisbon (Portugal) an invoice of goods amounting to 2576 milreas 250reas: required its value in federal money at $1.25 per milrea. Ans. $3220.3125.

5. A merchant in Bordeaux is indebted to a merchant of Baltimore 9275 francs 25 centimes; what is the amount in federal money, at 18¾ cts. per franc? Ans. $1739 11cts.

6. A. of New York draws on B. of St. Petersburg (Russia), for 2347 roubles 50 copecs, what is the value in United States currency, at 62½cts. per rouble?

Ans. $1467 18¾cts.

7. *Philadelphia*, 4 *mo.* 13, 1824.

Exchange 946 ducats 5 lire.

Thirty days after sight of this my first of exchange, (second and third of like tenor and date not paid,) pay to Burral and Co. nine hundred and forty-six ducats effective 5 lire, as per advice from

TIMOTHY FOLGER.

To Peter Lamotte, merchant, Venice.

What is the value of this bill at 74 cts. per ducat?

Ans. $700 50¼cts.

8. A bill of exchange being drawn upon Genoa, for 1576 scudi d'argento 7 lire 10 soldi; required the value in United States currency, at $1 55 per scudo.

Ans. $2443.9625.

9. A merchant in Holland wishing to remit 9405 florins currency, to Philadelphia, finds the agio 4½ per cent; and the exchange between Holland and Philadelphia, at 42 cts. per florin banco: quere the sum in money of the United States? Ans. $3780.

ARBITRATION OF EXCHANGE.

As the prices of bills of exchange depend on the fluctuations of commerce, and not entirely on the intrinsic value of the currencies of different countries, it sometimes happens, that drafts may be drawn, or remittances made, more advantageously by a circuitous than by a direct route. The calculation of the value of bills of exchange between two places, when effected through one or more intermediate ones, is called the Arbitration of Exchange.

RULE.

Calculate by two or more statings of the Rule of Three, or,

Set down in two columns, as antecedents and consequents, the quantities of the different currencies which are equal to each other, observing to make each consequent of the same denomination with the next following antecedent. Then multiply all the antecedents and all the consequents respectively together, and divide the products by their greatest common measure, the result will show the equivalent numbers, in denominations corresponding to the first antecedent and the last consequent. Hence the quantity of the one which is equivalent to any sum of the other, is easily determined.

EXAMPLES.

1. A merchant in Philadelphia imports from Petersburg a quantity of goods, amounting, per invoice, to 6765 roubles

50 copecs. Now, supposing Philadelphia exchanges with Petersburg at 60cts. the rouble; and with Bordeaux at 19cts. per franc, Bordeaux with Amsterdam at 7 francs for 3 florins, and Amsterdam with Petersburg at 9 florins for 7 roubles; which will be preferable, the direct or circuitous route, for the payment of this bill; and by how much?

6765.5 roub. at 60cts.=$4059 30cts. cost by direct route.
As 1fr. : 7fr. : : 19cts. : 133cts.=7 francs or 3 florins.
As 3flor. : 9flor. : : 133cts. : 399cts.= 9 flor. or 7 roubles.
As 7roub. : 6765.5roub. : : 399cts. : $3856.335 cost of circuitous route.
Hence the circuitous is preferable to the direct by $202.965.

antecedent. consequent.
Or thus, 19cts. =1 franc
7 francs =3 florins
9 florins =7 roubles

$$\frac{19\times7\times9}{3\times7}=\frac{19\times3\times21}{21}=57 \text{ or } 57\text{cts}=1 \text{ rouble}$$

Hence 6765.5 roubles at 57 cts.=$3856.335 as before.

3. E. of Leghorn being indebted to F. of Baltimore, 800 piastres, remits as follows: to Venice at 94 piastres for 100 ducats; thence to Cadiz at 1 ducat for 320 maravedies; thence to Lisbon 272 maravedies for 630 reas; thence to Amsterdam at 400 reas for 50d. flemish; thence to Paris at 56d. flemish for 3 francs; thence to London at 9 francs for 94d. sterling; and from London to Baltimore at 54d. sterling for $1. Required the rate of exchange, by this circuit, between Leghorn and Baltimore, and the number of dollars F. will receive.

Ans. { 612 piastres, worth $625.
F. receives $816 99cts.

3. If London remit £1000 sterling to Cadiz, by way of Amsterdam, at 35 shillings flemish for £1 sterling, thence to Nantz at 3 francs for 58d. flemish; thence to Cadiz at $8 for 31 francs; what is the rate of exchange between London and Cadiz resulting from this operation; and how many dollars at the latter place will be equivalent to the £1000 sterling?

Ans. £899 sterling=$5040, or $1=42¾d.+and £1000=5606dolls. 2 reals 9 maravedies.

4. P. of Philadelphia receives advice from his factor in

London, that he has purchased goods on his account to the amount of £1075 5s. in Bank of England notes; how much federal money will discharge the debt, bills of exchange being negotiated as follows:—Between Philadelphia and Bordeaux at 1 dollar for 5 francs; between Bordeaux and Hamburgh at 45 francs for 11 rix dollars; between Hamburgh and London at 1 rix dollar for 51d. specie; the price of silver per ounce being 6 shillings in Bank of England notes, and in specie 5s. 2d.?*

Ans. 3565 dollars.

5. My factor at Venice writes me word that he has shipped to New York on my account, goods valued per invoice, at 4945 ducats currency; how much federal money must I disburse on this account, exchange between New York and Cadiz, being 75 cents for one piastre, between Cadiz and Toulon 9 piastres for 43 francs; and between Toulon and Venice 10 francs for 3 ducats banco; the agio at 15 per cent? *Ans.* 2250 dollars.

As foreign weights and measures, particularly the French, are frequently mentioned in scientific works, the following brief account of a few of them is added for the information of the student.

Ancient French	foot, ($\frac{1}{6}$ of the toise)	=1.0657	English *ft.*
Rhinland	foot,	=1.0302	do.
Vienna,	foot of	=1.0367	do.
Amsterdam	do.	= .9263	do.
Portugal	do.	=1.0778	do.
Spain	do.	= .9267	do.
Sweden	do.	= .9747	do.
Russia	do.	=1.1617	do.
Moscow	do.	= .928	do.
Venice	do.	= 1.137	do.
Castile, vara of		=2.7447	do.
Russia, arsheen of		=2.3333	do.
France, metre of		=3.2815	do.

---

* During the time that the bank of England suspended its specie payments (from 1797 to 1823) the commercial transactions of that country were chiefly conducted by means of a paper currency, the value of which, compared with its nominal amount in spe-

Note.—The modern French measures are all derived from the metre, (or ten millionth part of the meridian from the pole of the earth to the equator) by decimal multiplication and division. The multiples are expressed by prefixing the Greek numerals *deca* (10) *hecto* (100) *kilo* (1000) and *myria* (10000) and the tenths, hundredths, thousandths, by prefixing the Latin terms *deci*, *centi*, *milli*, to the word metre. The value of the metre above given is the mean of the determinations of the French Academy, Kater in England, and Hassler in the United States.

Mile of Arabia		= 2148	English yards.
Bohemia		= 10137	do.
Denmark		= 8244	do.
Flanders		= 6869	do.
Germany, geographic		= 8101	do.
	long	= 10126	do.
	short	= 6859	do.
Holland		= 6395	do.
Hungary		= 9113	do.
Ireland		= 3038	do.
Italy		= 2025	do.
Lithuania		= 9781	do.
Poland	short	= 6075	do.
	long	= 8101	do.
Prussia		= 8468	do.
Rome, ancient		= 1612	do.
Verst of Russia		= 1167	do.
Mile of Scotland		= 1984	do.
Saxony		= 9905	do.
Silesia		= 7083	do.
Sweden		= 11700	do.
Switzerland		= 9153	do.
Westphalia		= 12151	do.
The pound weight of Denmark		= 1.1036*lb.*	avoirdupois.
Demarc of France		= 1.0794	do.
The pound weight of Holland		= 1.0893	do.

---

cie, is subject to frequent fluctuations. The student may hence understand, that the quotations of the price of silver, of the dollar, &c. meant the nominal value in bank paper, of the ounce of standard silver of the specie dollar, &c.

The pound weight of Naples . = .7071 lb. avoirdupois.
Poland (new) = .8909 do.
Portugal = 1.0121 do.
Spain (Castillian) = 1.0163 do.
Sweden = .9374 do.
Venice, *gross lb.* = 1.0556 do.
Vienna = 1.2340 do.
French gramme . = 0.00220653 or 15.44572*grs.* troy.

Note.—The gramme is an equipoise in vacuo to a cubic centimetre of distilled water reduced to its greatest density. The weights are formed from the gramme as the measures of distance are from the metre.

INVOLUTION,

OR THE RAISING OF POWERS.

Any number may be considered as a root, or that from which a power may arise.

The power of a number or root is the product of an unit continually multiplied by the given root, any proposed number of times.

The figure or quantity which indicates the number of multiplications thus made, is called the index of the power.

Thus $1\times5\times5\times5\times5=625=5^4$ or the 4th power of 5, hence $5^0=1$. Or when the index is 0 the power is always 1.

The second power is called the square; because it shows the area of a square whose side is the root of that power.

The third power is called the cube, because it shows the solidity of a cube, whose side is the root.

The first 9 powers of the nine digits are exhibited in the following table.

Roots.	Squares.	Cubes.	4th Powers.	5th Powers.	6th Powers.	7th Powers.	8th Powers.	9th Powers.
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

To involve a number to any given power.

RULE.

Multiply the number by itself as many times as there are units in the index less one.*

EXAMPLES.

1. Required the 5th power of 124.

---

* The product of any two powers of a root is a power of the same root whose index is the sum of the indices of the factors.

Thus $2^3 \times 2^5 = 2^8$ $3^4 \times 3^6 = 3^{10}$. Hence, in extensive involutions, we may frequently abridge the operations, by multiplying together two of the powers, which have been obtained, instead of repeating the multiplication with the root. We may also observe that, if we involve any two factors separately, to a given power, and take the product of those powers, we shall obtain the same power of the product of those factors: thus $5^4 \times 3^4 = 15^4$.

On this principle several of the examples above given are easily computed. In the 6th we may multiply the 8th power of 5 by the 8th power of 3 both taken from the table, for the power required. Also the 9th power of 9 multiplied by the 9th power of 3 will produce the answer to the 7th.

$124 \times 124 = 15376 = 124^2$ or 2d power of 124.
$15376 \times 124 = 1906624 = 124^3$ or 3d power of 124.
$1906624 \times 124 = 236421376 = 124^4$ 4th do. of 124.
$236421376 \times 124 = 29316250624 = 124^5$ 5th power.

2. What is the square of 549? *Ans.* 301401.
3. Required the tube of 54.36. *Ans.* 160634.321856.
4. What is the 5th power of 12.36?
Ans. 288464.4864562176.
5. What is the 6th power of 1.06?
Ans. 1.418519112256.
6. Required the 8th power of .15.
Ans. 0000002562890625.
7. Required the 9th power of 27.
Ans. 7625597484987.
8. What is the 20th power of 2? Ans. 1048576.
9. What is the 4th power of $\frac{2}{3}$? Ans. $\frac{16}{81}$.
10. What is the 5th power of $1\frac{3}{4}$? Ans. $16\frac{423}{1024}$.
11. Required the 7th power of $5\frac{2}{3}$.
Ans. $187626\frac{611}{2187}$.
12. Required the 9th power of $5\frac{2}{5}$?
Ans. $3904305\frac{1781862}{1953125}$.

EVOLUTION,

OR THE EXTRACTION OF ROOTS.

The extraction of roots is the finding of numbers which raised to given powers will be equal to given numbers.

Any power of a given number can always be found; but the roots of given numbers cannot be always exactly obtained; thus we cannot find a number which being multiplied by itself will produce the number 5. Yet the square root of 5 may be approximated, to any degree proposed, by means of decimals, as will be hereafter shown.

A root which cannot be accurately expressed in numbers is called a surd; thus, the square root of 5 is a surd.

The sign √ prefixed to a number denotes its root; thus

$\sqrt{5}$ denotes the square root of 5. $\sqrt[3]{8}$ indicates the cube root of 8. $\sqrt[4]{16}$ denotes the 4th root of 16.*

THE SQUARE ROOT.

To find the square root of a given number.

RULE.

Distinguish the given number into periods of two figures each, beginning at the decimal point, observing to annex a cipher to the decimals when their number is odd.

Take the greatest square contained in the left hand period, and subtract it from said period, placing its root in the quotient, and to the remainder annex the next period for a dividual.

Double the root already found for a defective divisor, and, reserving the units of the dividual, find the next quotient figure,† as in division; annex this figure to the defective divisor, then multiply by the last quotient figure, and subtract the product from the dividual. To the remainder bring down the next period, and proceed as before.‡

If a remainder should occur, the root may be continued at pleasure by annexing periods of ciphers to the given power.

The root of a vulgar fraction is found by reducing it to its lowest terms, and extracting the roots of the numerator and denominator, for the fraction required. Or, if either root be a surd, reduce the given fraction to decimals, and extract the root as in other cases.

---

* The sign $\sqrt{}$ is the *r* formerly used in England.

† The quotient found by this precept is sometimes too great. When this is the case it must be diminished, until the complete divisor multiplied by the quotient figure produces a result equal to or less than the dividual. A similar remark may be applied to the results obtained by the rules for the extraction of other roots.

‡ Defective divisors after the first, may be concisely found by adding the last figure of the root to the last complete divisor.

Note.—There must be as many integral and decimal places respectively in the root as there are periods of integers and decimals in the power.

PROOF.

Square the root found, adding the remainder, if any.

EXAMPLES.

1. What is the square root of 6324.728678 ?

$\dot{6}3\dot{2}4.7\dot{2}8\dot{6}7\dot{8}$	(79.5281628 Ans.
49	8261825.97
149)1424	5566971396
1341	715753465
	39764081
1585)8372	1590563
7925	636225
	7953
15902)44786	4771
31804	159
	63
159048)1298278	
1272384	6324.728676 proof.
15,9,0,5,6)25894	
15906	
9988	
9543	
445	
318	
127	
127	

When remainders occur, after obtaining more than half the proposed number of figures in the root, the process may be continued as in contracted division of decimals.

The proof, however, in such cases will not be quite accurate, as the final remainder does not appear.

2. What is the square root of 1679616. Ans. 1296.

3. Required the square root of 23097636 ? Ans. 4806.

4. What is the square root of 30138696025 ? Ans. 173605.

5. What is the square root of 5 ? Ans. 2.2360679.+

6. What is the square root of 2.96 ? Ans. 1.720465.+

7. Required the square root of 9.01. Ans. 3.0016662.+

8. Required the square root of $\frac{180}{245}$. Ans. $\frac{6}{7}$.

9. Required the square root of $2\frac{1}{4}$? Ans. $1\frac{1}{2}$.

10. What is the square root of $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{11}{12}$? Ans. .7756046.+

11. What is the square root of $9\frac{2}{7}$? Ans. 3.0472470+

Note.—The square of the hypothenuse or longest side of a right angled triangle is equal to the sum of the squares of the other sides.

12. A ladder, being placed at the distance of 12 feet from the bottom of an upright wall, is found to reach a window 16 feet from the ground ; what is its length ? Ans. 20ft.

13. The side of a square meadow is 325 yards, what is the length of the diagonal or line drawn from corner to corner ? Ans. 459.61942.

14. A tower being 45 feet high is surrounded by a ditch 60 feet wide, how long must a ladder be to reach from the outside of the ditch to the top of the tower ? Ans. 75 feet.

15. A ladder 50 feet long, being placed in the street, will reach a window 40 feet high, on one side, and without moving the foot will reach one 30 feet high on the other ; how wide was the street ? Ans. 70 feet.

16. A line of 79 feet in length extends from the top of a wall to a point 47 feet from its base ; what was its height? Ans. 63.498ft.+

THE CUBE ROOT.

To find the cube root of a given number.

RULE.

Distinguish the given number into periods of three figures each, beginning at the decimal point, completing the periods of decimals with ciphers, when necessary.

Under the left hand period set down the greatest cube it contains, putting the root in the quotient; subtract the cube, and to the remainder annex the next period for a dividual.

Take three times the square of the ascertained root for a defective divisor.

Reserve mentally the units and tens of the dividual, and try how often the defective divisor is contained in the rest. Place the result of this trial in the quotient, and its square to the right hand of the defective divisor, supplying the place of tens with a cipher, if the square be less than ten.

Multiply the last figure of the root by the rest, and by 30, the product added to the divisor before found, will make the complete divisor.

Multiply the complete divisor by the last quotient figure; subtract the product from the dividual; bring down the next period, and proceed as before.*

Note.—The last three precepts under the former rule, are equally applicable here.

PROOF.

Involve the root found to the third power, adding the remainder, if any, the result will be equal to the given power.

EXAMPLES.

1. What is the cube root of 955671625.?

---

* Defective divisors after the first, may be concisely found, thus, To the last complete divisor, add the number last used to complete it, and twice the square of the last figure of the root.

```
                                   . . .
                                   955671625.(985 An.
                               9³=729
                                   ------
9² × 3=243 def.div.24364 }   26524)226671          26524
      8×9×30=  2160      }         212192           2160
                 ----              ------            128
              2881225 }                            -----
                14700 } 2895925)14479625           28812
             --------           14479625           -----
                                --------
                   985
                   985
                  ----
                  4925
                 7880
                8865
               -------
                970225
                   985
              --------
               4851125
              7761800
             8732025
             ---------
             955671625 proof.
             ---------
```

2. What is the cube root of 236029032? Ans. 618.

3. What is the cube root of 1702209384?
Ans. 1194.

4. Required the cube root of 25803133875?
Ans. 2955.

5. Required the cube root of 520882881.336?
Ans. 804.6.

6. What is the cube root of 119.625?
Ans. 4.92728.+

7. What is the cube root of 331.373888?
Ans. 6.92.

8. What is the cube root of 5? Ans. 1.709976.+

9. What is the cube root of $\frac{729}{1331}$? Ans. $\frac{9}{11}$.

10. Required the cube root of $\frac{29160}{34295}$. *Ans* $\frac{18}{19}$.

11. What is the cube root of $15\frac{5}{8}$? Ans. $2\frac{1}{2}$
12. Required the cube root of $\frac{2}{3}$? Ans. 87358.+
13. Required the cube root of $\frac{5}{9}$? Ans. 822070.+
14. What is the cube root of $\frac{15}{16}$? Ans. 978717.—
15. What is the cube root of $7\frac{3}{5}$? Ans. 1.966095.+

In the extraction of surd roots, when they are required to extend to a great number of decimals, the process may be shortened by various methods, of which the following is one of the most convenient.

Find the first two or three figures of the root as before, then,

Involve the root thus found to its cube, and multiply by 3 for a dividend. Subtract said cube from the given number for a divisor; divide and reserve the quotient.

To three times the reserved quotient, add 2 for a dividend.

To this dividend add 3, and multiply the sum by the reserved quotient for a divisor. To the quotient, thence arising, add 1, and multiply the sum by the ascertained root, for the true root nearly.

Involve the root thus found, and proceed as before. By this method, the number of places may be generally tripled at each operation.

To find, by this method, the cube root of 16.

The first two figures found in the usual way are 2.5

$\overline{2.5}|^3 = 15.625$, $15.625 \times 3 = 46.875 =$ dividend.

$16 - 15.625 = .375 =$ divisor.

$$\frac{46.875}{.375} = 125. = \text{reserved quotient.}$$

$125 \times 3 = 375$, $375 + 2 = 377 =$ dividend.

$377 + 3 = 380$, $380 \times 125 = 47500 =$ divisor.

$$\frac{377}{47500} = .007936, \quad 1.007936 \times 2.5 = 2.51984 = \text{the required root, true to 6 places.}$$

By repeating the process with this result, the root may be found to 18 or 19 places.

What is the cube root of 43? Ans. 3.503398060211.

Required the cube root of 344. Ans. 7.006796120422.

TO EXTRACT THE ROOTS OF POWERS IN GENERAL.

RULE.

If the index of the given power be divisible by 2 or 3, extract the square or cube root of the given number; the result will be a power of the required root one half or one third as high as the given one. Hence when the index of the given power is composed of the factors 2 and 3, or of powers of either, the required root may be obtained by the rules already given.

But when the index of the given power is not a multiple of 2 or 3, distinguish the given number, beginning at the decimal point, into periods of as many figures each, as there are units in the given index.

Find by the table of powers, or by trial, the greatest power contained in the first left hand period; set its root in the quotient, subtract the power from the period, and to the remainder bring down the next figure of the given number for a dividual.

Involve the ascertained root to the power whose index is one less than the given one, and multiply by the given index for a divisor. Divide, and set down the quotient as the next figure of the root.*

Involve the whole ascertained root to the given power, subtract from the first two periods, and proceed as before for the several figures of the root.

EXAMPLES.

1. Required the 5th root of 184528125.

$4^4 = 256$ 184528125(45

5 1024

Divisor 1280)8212

$45^5 =$ 184528125

---

* See note, page 144.

The quotient first found is 6, but 46 involved to the 5th power proves too great. The quotient is therefore diminished to 5.

2. What is the 6th root of 30840979456? Ans. 56.
3. Required the 7th root of 373669453125. Ans. 45.
4. Required the 9th root of 4. Ans. 1.16653.

As this method, when the root is to be found to a great number of places, is exceedingly tedious, the following may be adopted.

Find the first two figures of the root, as above directed, which call the ascertained root.

Involve the ascertained root to the given power, and multiply by the given index for a dividend.

Subtract the power of the ascertained root from the corresponding periods of the given number, for a divisor. Divide and reserve the quotient.

To 6 times the reserved quotient add the given index increased by a unit for a second dividend.

To 6 times the reserved quotient add 4 times the given index, subtract 2 from the sum, and multiply by the reserved quotient for a second divisor. Divide, add 1 to the quotient, and multiply by the ascertained root for the true root nearly. By repeating the process, the root may be obtained to any degree of accuracy proposed. In surd roots, the number of places may be generally tripled, by this method, at each operation.

Or, to those acquainted with the language of Algebra, the following method will be found convenient.

Let N= the given number whose root is required.
n= the given index.
r= the root found as before to two or three figures.

$$x=\frac{N-r^n}{N+r^n}$$

$$y=\frac{2x+\frac{2}{3}x^3+\frac{2}{5}x^5+\frac{2}{7}x^7+\&c.}{n}$$

$$z=1+y+\frac{y^2}{2}+\frac{y^3}{6}+\frac{y^4}{24}+\frac{y^5}{120},\ \&c.$$

then rz= the true root very nearly.

5. Required the 5th root of 5764272.

By the first method.

The first two figures of the root, found by the general rule, are 22.

given number = 5764272
22^5 = 5153632

divisor = 610640

5153632
5

25768160 = dividend

$$\frac{25768160}{610640} = 42.1985 = \text{reserved qnotient.}$$

42.1985
6

253.191
6

259.191 = second dividend

253.191 = 6 times res. quotient
18 = 5×4—2.

271.191
42.1985

11444.218 = second divisor

$$\frac{259.191}{11444.218} = .0226482.$$

1.0226482
22 = ascertained root.

22.4982604 = root required, which is correct to 5 places of decimals

By the second method.

$$\begin{array}{rr}
\text{Given number} & 5764272 \\
22^5 = & 5153632 \\
\hline
\text{sum} & 10917904 \\
\hline
\text{difference} & 610640 \\
\hline
\end{array}$$

$$\begin{array}{r}
\frac{610640}{10917904} = .055930015 = x \\
58371 = \frac{1}{3}x^3 \\
109 = \frac{1}{5}x^5 \\
\hline
.055988495 \\
2 \\
\hline
5).111976990 \\
\hline
.022395398 = y. \\
\hline
\end{array}$$

$$\begin{array}{rl}
1. & = 1 \\
.022395398 & = y \\
2507776 & = \frac{1}{2}y^2 \\
1870 & = \frac{1}{6}y^3 \\
9 & = \frac{1}{24}y^4 \\
\hline
1.022648053 & = z \\
22 & = r \\
\hline
22.498257166 & = \text{root required.} \\
\hline
\end{array}$$

The same as before very nearly.

6. Required the 365th root of 1.06.
Ans. 1.0001596535.

7. Required the 7th root of 14. Ans. 1.4579162496.

8. What is the 15th root of 1.0075?
Ans. 1.00049825841.

DUODECIMALS.

Duodecimals are fractions having 12 or some power thereof for their denominators. They are chiefly employed in computing surfaces and solidities, where the dimensions are taken in feet and inches.

The feet are considered as the units, and the parts are usually denoted by placing the characters of their denominations over them.

The denominations are,—

1 foot (*ft.*) = 12 inches, *in.*
1 inch = 12 seconds, ″
1 second = 12 thirds, ‴ &c.

ADDITION AND SUBTRACTION OF DUODECIMALS.

RULE.

Set feet under feet, inches under inches, &c. then proceed as in Compound Addition or Subtraction.

EXAMPLES.

ft.	*in.*	″	
14	6	4	
53	10	7	
27	9	11	
36	7	8	
48	6	9	
181	5	3	*sum.*

ft.	*in.*	″	
18	6	3	
29	11	4	
43	9	7	
56	6	3	
74	10	5	
			sum.

ft.	*in.*	″	
43	9	7	
78	11	3	
83	9	6	
71	11	5	
83	7	4	
			sum.

	ft.	*in.*	″
From	76	8	4
Take	37	11	5
Rem.	38	8	11

ft.	*in.*	″
176	8	9
98	6	11

ft.	*in.*	″	‴
279	11	3	4
127	10	7	9

From the sum of 25*ft.* 4*in* 3′, 45*ft.* 7*in.* 6″, and 17*ft.* 7*in.* 8″, take the sum of 18*ft.* 7*in.* 8″, 11*ft.* 4*in.* 9″, and 8*ft.* 6*in.* 6″. *Rem.* 50*ft.* 0*in.* 6″.

MULTIPLICATION OF DUODECIMALS.

RULE.

Set the multiplier in such order that the feet thereof may stand under the lowest denomination of the multiplicand. Multiply, carry one for every 12 from one denomination to another, and set down the result so that the lowest denomination in the product may stand under the corresponding multiplier. Then add as in Addition. Or, Multiply by the number of feet and take aliquot parts for the inches, &c.*

EXAMPLES.

A load of bark measures 12*ft.* 3*in.* long, 4*ft.* 2 *in.* high, and 3*ft.* 6*in.* wide. Required the number of solid feet therein.

Note.—When the feet in the multiplier exceed 12, use the component parts as in Compound Multiplication.

$$\begin{array}{rrrrr} & ft. & in. & & \\ & 12 & 3 & & \\ & & 4 & 2 & \\ \hline & 49 & 0 & & \\ & 2 & 0 & 6 & \\ \hline & 51 & 0 & 6 & \\ & & & 3 & 6 \\ \hline & 153 & 1 & 6 & \\ & 25 & 6 & 3 & 0 \\ \hline \text{Ans.} & 178 & 7 & 9 & = 1 \text{ cord } 50ft.\ 7\ in.\ 9''. \\ \hline \end{array}$$

$$\begin{array}{rrrr} & ft. & in. & \\ \text{Or, } 2|\tfrac{1}{6}) & 12 & 3 & \\ & & 4 & \\ \hline & 49 & 0 & \\ & 2 & 0 & 6 \\ \hline 6|\tfrac{1}{2}) & 51 & 0 & 6 \\ & & & 3 \\ \hline & 153 & 1 & 6 \\ & 25 & 6 & 3 \\ \hline ft. & 178 & 7 & 9'' \\ \hline \end{array}$$

---

* Though feet are here *sometimes* said to be multiplied by *feet*, *inches*, &c. yet, the subject being duly considered, it will appear that the multipliers are still abstract numbers, integral or fractional.

2. A board measures 28*ft.* 10*in.* long, and 3*ft.* 5*in.* wide; required its content. *Ans.* 68*ft.* 6*in.* 2″.

3. A partition measures 82*ft.* 9*in.* by 15*ft.* 6*in.* required its content. *Ans.* 1282*ft.* 7*in.* 6″.

4. A merchant imports from London 6 bales of goods of the following dimensions, viz.

	length.		*width.*		*height.*	
No 1.	2*ft.*	10*in.*	2*ft.*	4*in.*	1*ft.*	9 *in.*
2.	2	10	2	6	1	3
3.	3	6	2	2	1	8
4.	2	10	2	8	1	9
5.	2	10	2	6	1	9
6.	2	11	2	8	1	8

Required the solid feet in the whole, and the cost of the freight, at $20 a ton; 40 solid feet being supposed to weigh a ton?

Ans. { Content 71*ft.* 7*in.*
Cost of freight $35 79cts.

TONNAGE OF SHIPS.

The burden of a ship, as a weight, is ascertained by the depth of water that she draws. On the principles of hydrostatics, the weight of any floating body is equal to that of the mass of fluid displaced by it; ane the weight of a ship's burden is the difference between the column of water drawn by her, when in ballast, and when laden.

To ascertain a ship's legal tonage.

If the ship be double decked, take the length thereof from the fore part of the main stem, to the after part of the stern post, above the upper deck, the breadth thereof at the broadest part above the main wales, half of which breadth shall be accounted the depth of the vessel; then deduct from the length ⅗ of the breadth, multiply the remainder by the breadth, and the product by the depth, divide by 95, the quotient is deemed the tonnage of such a vessel. If the ship be singled decked, take the length and breadth, as above directed, deduct from the length $\frac{3}{5}$ of the breadth, take the depth from the under side of the deck plank to the ceiling in the hold, and multiply and divide as aforesaid for the tonnage. [*Laws of the United States*, chap. 128, Sec. 64.] Though this act does not indicate by what specific measures this length, breadth, and depth are to be taken, yet it has always been understood here to be in feet and tenths of a foot. [*Adam's Report on Weights and Measures*, page 65.]

EXAMPLES.

Required the legal tonnage of a double decked vessel, the length being 110.5 feet and the breadth 30.6 feet?

$\frac{3}{5}$ of 30.6 = 18.36

feet.
110.5
18.36

92.14
30.6 breadth,

2819.484
15.3 depth,

95)43138.1052

454.085 tons.

2. What is the legal tonnage of a single decked ship, which measures 76.4 feet in length, 28.6 feet in breadth, and 12.3 feet in depth? Ans. 212.327.

POSITION, OR THE RULE OF FALSE.

Position is a rule for determining, by means of supposed numbers, an unknown quantity from which a given quantity is obtained by an assigned process.

SINGLE POSITION.

Single Position includes the solution of questions by one supposed number.

RULE.

Assume any number and perform on it the operation indicated in the question: then,

As the result of this operation
Is to the given result,
So is the assumed number
To the number sought.

PROOF.

Perform, on the number found, the operation indicated in the question, the result will be the given number.

Note.—If two supposed numbers be substituted for the unknown one, and the results bear the same ratio to each other, as the supposed numbers themselves, the question may be solved by Single Position.

EXAMPLES.

1. Three unequal vents will severally empty a vessel of 120 gallons in 1 hour, 2 hours, and 3 hours: in what time will they discharge it if all run at once?

Suppose 30min. galls
Then the 1st will discharge 60
2nd 30
3rd 20
—— galls. min. min.
110 : 120 : : 30 : 32$\frac{8}{11}$.

Or, without adverting to the contents of the vessel,

The first vent, in 30 min, will discharge $\frac{1}{2}$, the second $\frac{1}{4}$, and the third $\frac{1}{6}$.

m. m.
$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{6}{12}+\frac{3}{12}+\frac{2}{12}=\frac{11}{12}$. As $\frac{11}{12}$: 1 : : 30 : 32$\frac{8}{11}$. *A.*

2. What sum is that of which $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, make $228? Ans. $240.

3. Of a certain contribution, A. pays $\frac{1}{3}$, B. $\frac{1}{5}$, C. $\frac{1}{4}$, and D. the rest, which is \$26, required the whole amount? Ans. \$120.

4. A. and B. have the same income. A. saves $\frac{1}{5}$ of his yearly; but B. by spending as much in 3 years as A. does in 4, finds himself at the end of five years, \$200 in debt; quere the income? Ans. \$600.

DOUBLE POSITION.

Double Position is used for solving such questions as require two supposed numbers.

RULE.

Suppose two numbers, and perform on each of them the operation indicated by the question, and note the errors of the results. If the supposed numbers are both too great or both too small, which can always be known by examining the nature of the question, and the results of the operations, the errors are said to be alike; but if one supposed number is too great, and the other too small, the errors are said to be unlike.

Multiply each supposed number by the error resulting from the other. Then if the errors are $\left\{\begin{matrix}\text{alike}\\\text{unlike}\end{matrix}\right\}$ divide the $\left\{\begin{matrix}\text{difference}\\\text{sum}\end{matrix}\right\}$ of the products by the $\left\{\begin{matrix}\text{difference}\\\text{sum}\end{matrix}\right\}$ of the errors, for the number sought.

Or, suppose two numbers, and perform on each the operation indicated in the question. Take the difference of the results, and the difference between one of them and the given result; then,

As the former of these differences
Is to the latter,
So is the difference of the supposed numbers,
To the difference between the number sought and the supposed number producing the latter difference.

PROOF.

As in Single Position.

Note.—If three supposed numbers be successively substituted for the unknown one, and the differences of the results bear the same ratio to each other as the differences of the supposed numbers, the question may be solved by this rule. If those differences are not proportional, this method can produce, at most, only an approximation to the truth.

EXAMPLES.

1. Divide $1000 so that B. may have twice as much as A. wanting $80, and C. three times as much, wanting $150: quere the share of each?

		$	$	
	Suppose	A. 200	180	
	then	B. 320	280	
	and	C. 450	390	
		970	850	
		1000	1000	
Error, supposed numb. too small.		30	150	Error, supposed numb. too small.

$	$	$
150	200	180
30	150	30
120	30000	5400
	5400	

120)2460

205 A's share.
330 B's do.
465 C's do.

Proof. 1000 Whole sum.

	$		$
Or, suppose A.	200	——	180=20
then B.	320		280
and C.	450		390
	970		850
	1000		970
	30		120

As 120 : 30 : : 20 : 5
200

205 A's share as before.

2. A. B. C. and D. possess an undivided estate, worth 12,000 dollars, of which B. claims 400 dollars more than A. C. 200 dollars more than B. and D. 400 dollars more than C.; what is the claim of each?
Ans. A. $2500, B. $2900, C. $3100, D. $3500.

3. A. is 20 years of age, B's age is A's and half C's, and C's age is equal the sum of the other two. Required the ages of B. and C. Ans. B. 60, C. 80.

4. A. and B. put equal sums in trade. A. gained a sum equal to ¼ of his stock, and B. lost $450, when A's money was double of B's. What was the sum laid out by each?
Ans. $1200.

5. Two persons comparing their ages, find them, at present, in the ratio of 7 to 5, but that 30 years ago they were as 2 to 1; what is now the age of each? Ans. 70 and 50.

6. The ages of a man and his wife, at the time of marriage, were to each other as 4 to 3; but now at the end of 35 years they are as 15 to 13, what were their ages at the former period? Ans. 40 and 30 yrs.

Such questions as can be *accurately* solved by this rule, admit of very simple solutions by Algebra. The second rule is frequently used to approximate the results in cases which do not admit of accurate solutions.

Though these rules are generally given by writers on practical arithmetic, it is proper to remember that they are not a necessary part of the science, and, indeed, the accomplished arithmetician will seldom or never resort to them. The 4th example page 159 may be solved thus: From the data it is obvious that A. spends $\frac{4}{5}$, and B. $\frac{4}{5}\times\frac{4}{3}=\frac{16}{15}$ of the yearly income in a year. Hence B. sinks annually a sum equal to $\frac{1}{15}$ of his income. But he goes in debt $40 a year, therefore $40 × 15 = $600 = the income required.

Again, the 6th example, page 161, may be readily solved without the aid of this rule.

From what is given we perceive that the difference of their ages is $\frac{2}{15}$ of his present age, and the same difference is $\frac{1}{4}$ of his age at the time of marriage; hence that difference multiplied by 4 and $\frac{15}{2}$ respectively, will produce his age at the two periods in question; consequently $\frac{15}{2}$ — 4 : 4 :: 35 : 40, the age required.

PROMISCUOUS EXAMPLES.

1. How many bushels of wheat, at $1 25cts. per bushel, can be purchased for $946 25cts ? Ans. 757 bu.

2. How many dollars, half dollars, quarter dollars, and dimes, of each an equal number, may be coined at the United States' mint out of 305 lb. 3oz. troy weight of pure silver ? Ans. 2560.

3. About the beginning of Edward VI.'s reign, the pound troy of standard gold, containing 10oz. of pure gold, was reckoned in coins at £30 sterling. In the time of George III. the lb. troy of standard gold, containing 11oz. of pure gold, was coined into 44½ guineas, each reckoned at 21 shillings sterling. What was the value, in gold coin of the latter period, of the gold £ sterling of the former one ? Ans. £1.415909.

4. At the time first mentioned in the former example, the lb. troy of standard silver, containing 4oz. of pure silver, was coined into 48 English shillings, in 1816 the lb. troy of standard silver, containing 11oz. 2dwts. of pure silver, was coined into 66 shillings at the English mint. Quere the value of the silver shilling of the former period in silver shillings of 1816 ? Ans. $\frac{55}{111}$s.

5. In the English mint the lb. of standard gold, one ounce of which is alloy, is reckoned at £46 14s. 6d., and

the lb. of standard silver, 18dwts. of which are alloy, is reckoned at £3 6s.; how many lbs. of pure silver are equivalent, there, to 1lb. of pure gold? Ans. 14.287809lbs.

6. If I remit 50000 federal dollars, of the standard weight and purity, to Great Britain, obtain their equivalent in English guineas, and exchange those guineas at the United States' mint for their value in dollars, how much do I gain or lose by the transaction, supposing no expense incurred by the transportation? Ans. $2492 30cts. gained.

7. A. B. C. and D. are sharers in a parcel of merchandise; A. B. and C. have $350, B. C. and D. $345, C. D. and A. $400, and D. A. and B. $378; required the whole sum and each partner's share.

Whole sum $491, A. $146, B. 91, C. $113, D. $141. Ans.

8. If a stationer by selling quills at $2 16cts. per thousand, clear $\frac{1}{3}$ of the money received; what will be the gain per cent if he raise the price to $2 50cts. per thousand? Ans. $73 61cts.

9. At what time, between 4 and 5 o'clock, are the hour and minute hands of a watch exactly together?

Ans. 21$\frac{9}{11}$min. past 4.

10. What difference is there between the interest of $350 for 8 years at 4 per cent per annum; and the discount of the same sum at the same rate and for the same time?

Ans. $27 15$\frac{5}{33}$cts.

11. A gentleman left his son a fortune, $\frac{5}{16}$ of which he spent in 3 months, $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder lasted him 9 months longer, when he had only $2640 left, what sum did the father bequeath him? Ans. $10240.

12. A. can mow an acre of grass in 7$\frac{1}{3}$hrs. B. in 8$\frac{4}{5}$hrs. in what time will they jointly perform it? Ans. 4hrs.

13. What is the sum of $\frac{1}{3}$, $\frac{1}{2}$ of $\frac{1}{3}$, and $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{6}$ of 5 dollars? Ans. 4\frac{1}{6}$.

14. I sum up half mankind,
And add two thirds of the remaining half,
And find the total of their hopes and fears,
Dreams, empty dreams. *Cowper.*

Quere. What part of mankind are mere dreamers, according to this author? Ans. $\frac{1}{6}$.

15. When a mercurial barometer stands at 2.5 feet

what would be the height of a homogeneous atmosphere,* considering the gravity of mercury to that of air at the earth's surface as 13600 to $1\frac{2}{9}$; the heights of equiponderant columns of different fluids being reciprocally as their gravities? Ans. $27818\frac{2}{11}$ feet, or 5.268 miles.

16. Two travellers, A. and B. at the distance of 59 miles from each other, set out in order to meet. A. begins his journey 1 hour before B. and travels 7 miles in 2 hours; B proceeds 8 miles in 3 hours. How far will they travel respectively before they meet?

Ans. A. 35 miles, B. 24.

17. A market woman having bought a quantity of apples, at the rate of 250 for a dollar, and sold one half of them at 2 for a cent, and the remaining half at 3 for a cent, finds she has gained 10 cents by the transaction. How many did she buy? Ans. 600.

18. A trader having increased his estate annually 10 per cent. for 7 years, finds himself worth $19487.171, what was his capital at the commencement of the term?

Ans. $10000.

19. A hare starts up 50 of its own leaps before a greyhound, and takes 4 leaps while the hound takes 3; but the hound goes as far at 2 leaps as the hare does at 3; how many leaps will the hound take to catch the hare?

Ans. 300.

20. In a shoal of herrings, 7 miles long, $2\frac{1}{2}$ in breadth, and 350 yards deep, how many herrings, allowing 150 to a solid foot; and how many casks, each containing 900 herrings, would they fill?

Ans. 7683984000000 herrings, and 8537760000 casks.

21. A, B and C purchase a vessel in company; A pays $\frac{2}{5}$ of the price, B $\frac{3}{7}$, and C the rest, which was $900. How much did A and B severally pay, and what part had C?

Ans. A $2100, B $2250, C $\frac{6}{35}$.

22. The inhabitants of London are supposed to consume annually, 98245 beeves, which may be estimated at 6 gui-

---

* A homogeneous atmosphere is an atmosphere supposed to be of the same weight as that which surrounds the earth, and whose density is every where equal to that of the air at the surface of the earth.

neas each; 711124 sheep and lambs, at 10 shillings each; 194770 calves, at 23 shillings each; 186934 hogs, at 45 shillings each; 1398 boat-loads of fish, at 3 guineas a load; 16000000 lbs. of butter, at 8d a lb.; 20000000 lbs. of cheese, at 6d. a lb.; 7000000 gallons of milk, at 8d. a gallon; 1172494 barrels of strong beer, at 36 shillings each. What is the amount of all these expenditures, the shillings being supposed to be the silver shillings of 1816?

Ans. $21882683 32.

23. A man who was but slightly versed in numbers, bequeaths his estate, valued at 6500 dollars, to his wife and two children, as follows: to his wife, $\frac{1}{2}$; to his eldest son, $\frac{1}{3}$; and to the other, $\frac{1}{4}$. How much must they severally receive, so as to fulfil the design of the testator?

Ans. Wife, $3000, eldest son, $2000, younger son, $1500.

24. A man bequeaths $\frac{1}{3}$ of his estate to his widow; $\frac{2}{3}$ of the remainder to his son, and $\frac{1}{3}$ to his daughter; but the son, generously agreeing to make his sister's share equal to his own, pays her, for this purpose, 2000 dollars out of his portion. What was the whole estate left? Ans. $18000.

25. A, B and C can complete a piece of work in 15 days; A can do it in 30 days, and B in 40. In what time can C perform it? Ans. 120 days.

26. A stone let fall from the top of a cliff, is found to reach the base in 5 seconds. What was the height?*

Ans. 302 ft. 1 in.

27. A ball projected directly upwards, being found to return to the earth in 12 seconds, required the height of its ascent, upon the supposition that the ascending and descending velocities were equal. Ans. 579 feet.

28. What is the least number, which, being divided by 6, 7, 8, 9, 10, and 12, shall always leave a remainder of 5?

Ans. 2525.

29. Required the least number, which, being divided by each of the nine digits, shall leave no remainder; but when divided by 17, the remainder shall be 3. Result, 12600.

30. A poor woman carrying eggs to market, met with a

---

* Heavy bodies, falling from a state of rest, descend through spaces proportional to the squares of the times; and the distance fallen during the first second, is $16\frac{1}{12}$ feet, very nearly.

drunken man, who broke them. But upon coming to his senses, and reflecting upon what he had done, he offered to pay for them, enquiring, at the same time, how many there were. She replied she knew the number was less than 400, and that when counted out by 2, 3, 4, 5, or 6 at a time, there was still one left, but when counted by 7 at a time, there was none left. How many were there? Ans. 301.

31. A servant having eloped from his master travels 14 hours in the day at the rate of 3½ miles an hour; at the end of two days a courier is sent in pursuit, who travels 9 hours in the day, at the rate of 7 miles an hour. In what time will he overtake him? Ans. 7 days.

32. Divide 1100 dollars among A, B and C, in such manner that B shall have 100 dollars more than A, and C 150 more than B; and assign the share of each.

Result, A 250, B 350, C 500.

33. The ages of a man and wife, at the time of marriage, were in the ratio of 7 to 6; and at the end of 30 years they are found to be as 17 to 16. Required their ages at the former period. Ans. 21 and 18.

34. A Greek epitaph designed for the tomb of Diophantus, is said to have stated that he passed $\frac{1}{6}$ of his life in childhood; $\frac{1}{12}$ in adolescence; that after $\frac{1}{7}$ and 5 years more had been passed in a married state, he had a son who lived to ½ his own age, and whom he survived 4 years. What then was the age of Diophantus? Ans. 84 years.

35. A farmer having hired a labourer on condition that for every day he wrought he should receive 50 cts., and for every day he was idle he should forfeit 20 cts., finds at the end of 420 days that neither is indebted to the other. How many days did he work? Ans. 120.

36. A man and his wife usually drank a cask of beer in 12 days; but when the man was from home it lasted the woman 30 days. How long would the man alone be in drinking it? Ans. 20 days.

37. A sum of money being divided among 6 poor persons, the second received 10d, the third 14d, the fourth 25d, the fifth 28d, and the sixth 33d less than the first. The whole sum divided was 10d more than three times what the first received. How much did they severally receive?

Ans. 40d, 30d, 26d, 15d, 12d and 7d.

38. A mercer having cut 19 yds. from each of 3 equal pieces of silk, and 17 yds. from another of the same length, finds the four remnants together measure 142 yards. What was the original length of each piece? Ans. 54 yards.

39. A grazier having two flocks of sheep, consisting of equal numbers, sells 39 from one, and 93 from the other, and then finds one flock twice as numerous as the other. What number did each contain at first? Ans. 147.

40. From each of 16 coins, an artist filed the value of 20 cents; when the coins being examined were found worth only $11 68 cts. What was the original value of each? Ans. 93 cents.

41. There are two numbers whose sum is to their difference as 8 to 1; and the difference of whose squares is 128. What are the numbers? Ans. 13 and 14.

END OF PART I.

ok